Homework 6: Sandpiles
02-201: Programming for Scientists
Due: Thursday, March 17, 2016 at 11:59 PM

1. Set up

1. Inside of your `src` directory, create a directory called `sandpile`.
2. Copy the `canvas.go` file from HW4 and the `gifhelper.go` file from HW5 into the `sandpile` directory. Also make sure that you still have the `code.google.com` and `gogif` directories in your `src` directory.
3. Set your GOPATH environment variable to the location of your `go` directory that you made above. On a Mac, use the command

   ```
   export GOPATH=/Users/pcompeau/Desktop/go
   ```

   where you replace the directory name after the `=` with the location of the `go` directory you just made.

   On Windows, use the command

   ```
   set GOPATH=C:\Users\pcompeau\Desktop\go
   ```

2. Assignment

2.1 Sandpiles

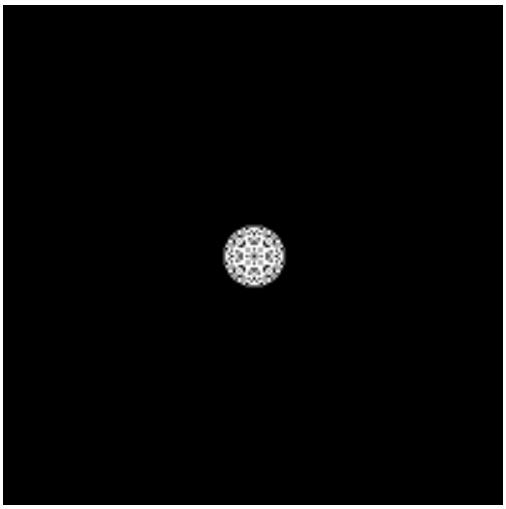
Imagine a huge 2-D checkerboard on which you place piles of coins of various heights on some of the squares (at most 1 pile per square). Consider the following toppling operation:

> topple(r, c): if square (r, c) has ≥ 4 coins on it, move 1 coin from (r, c) to each of the 4 neighbors of (r, c) (diagonal neighbors don't count, only north, south, east, and west). If square (r, c) has < 4 coins, do nothing.

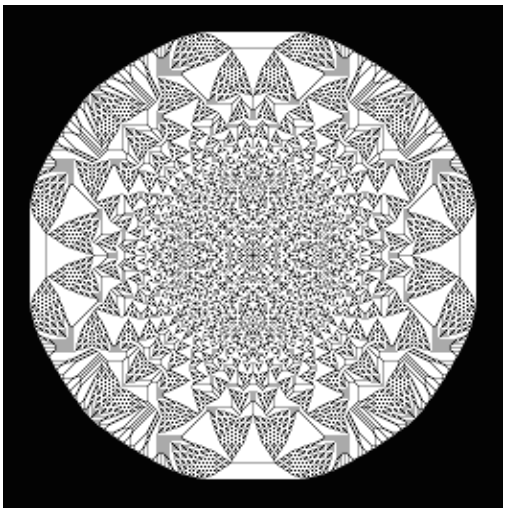
(**Important Note:** if (r, c) lies on the boundary of the board, model the topple operation as if the coins fall off the edge of the board. In other words, you do not have to keep track of the coins that fall off, but you should remove four coins from (r, c) even if (r, c) lies on the boundary of the board. This is important, because if you don't account for this assumption, there will be some inputs that cause your program to run forever.)

A configuration of coins is said to be **stable** if all squares have < 4 coins on them (i.e., there are no more topple operations to perform). If we repeatedly topple until we can't topple any more, then we will eventually end up at a stable configuration. There is a (surprising!) theorem stating that for a given initial configuration, the order of the topples that you perform won't affect the stable configuration that you reach.

For example, say that you start with a pile of 1000 coins on a 200×200 board. Then you will obtain the following stable board (zoom in to see the individual pixels):



If you instead start with 100,000 coins on a single square, and no coins elsewhere, then you will end up with the following configuration for a significantly large board:



where the color indicates the number of coins (0=black, 3=white, and 1 and 2 are intermediate shades of gray).

You can read more about these sandpiles here:

`http://www.cmu.edu/homepage/computing/2014/fall/lifes-a-beach.shtml`

2.2 What you should do

Write a program that can be run with the following command line:

```
sandpile SIZE PILE
```

where `SIZE` and `PILE` are both positive integers. `SIZE` gives the size of checkerboard which will be `SIZE` $\times$ `SIZE`. `PILE` gives the number of coins that are to be placed on the middle square at position ($\lfloor$`SIZE`$/2\rfloor$, $\lfloor$`SIZE`$/2\rfloor$) at the start.

This program should find the stable configuration associated with the given initial configuration. It should then create an animated GIF where each frame of the GIF represents one iteration of the board, and then draw the final board in a PNG file called `board.png`. The colors corresponding to each number of coins should be, given in (Red, Green, Blue) values:

0	(0,0,0)
1	(85,85,85)
2	(170, 170, 170)
3	(255, 255, 255)

Each board square should be drawn as a single pixel (i.e., a 1×1 square).

Your program must create a new type called `Board`, with the following methods:

- `Topple(r, c int)`: topples (r, c) until it can't be toppled any more.
- `Contains(r,c int) bool`: returns `true` if (r, c) is within the field and `false` otherwise.
- `Set(r,c, value int)`: sets the value of cell (r, c).
- `Cell(r, c int) int`: returns the value of the cell (r, c).
- `IsConverged() bool`: returns `true` if there are no cells with ≥ 4 coins on them and `false` otherwise.
- `NumRows() int`: returns the number of rows on the board.
- `NumCols() int`: returns the number of columns on the board.

Speed. Your program should be fast enough to run `./sandpile 200 10000` in at most a few seconds. It should be able to run `./sandpile 2000 100000` in at most 15 minutes (give or take a factor of 2 depending on your computer). However, please keep the following quotation in mind:

"*About 97% of the time: premature optimization is the root of all evil.*" — D. Knuth

In other words, it is not a bad idea to start with a working program that works on smaller inputs and then work on ways to speed it up on larger inputs. To test the speed of a function, you may find the benchmarking provided by the `testing` package useful (see `http://tinyurl.com/pcacykp` for details).

3. Learning outcomes

After completing this assignment, you should have

- earned more experience with an "object-oriented" way of thinking
- learned about sandpiles
- worked on optimizing a program faster if needed