

Phillip Compeau @PhillipCompeau



This is the best video to introduce a computational biology course. There will never be a better one



3:02 PM · Jun 28, 2023 · 536.3K Views III View post engagements **1** 822 ♡ 3,118 ₾ **Q** 74 707

Assembling Genomes

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ENERNIN

WENTER MANAGEM

Eternity II: The Highest-Stakes Puzzle in History



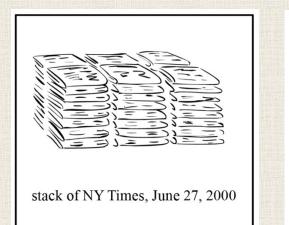
Courtesy: Matej Bat'ha

AN INTRODUCTION TO GENOME SEQUENCING



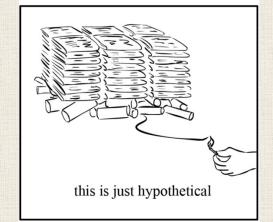


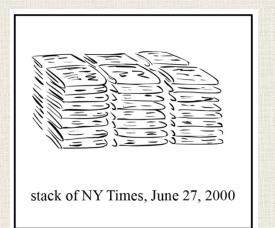


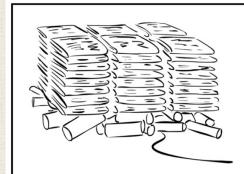




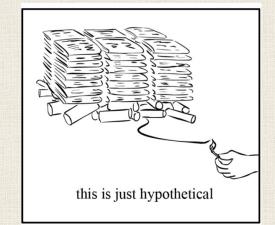
stack of NY Times, June 27, 2000 on a pile of dynamite



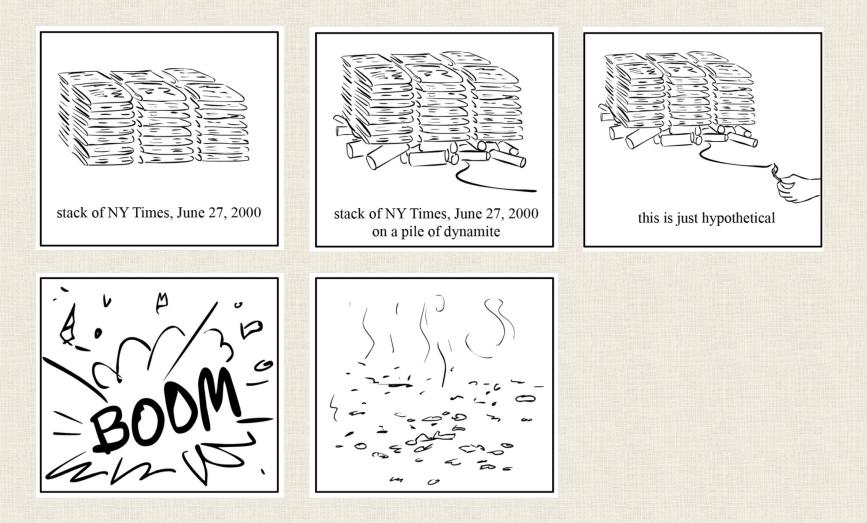


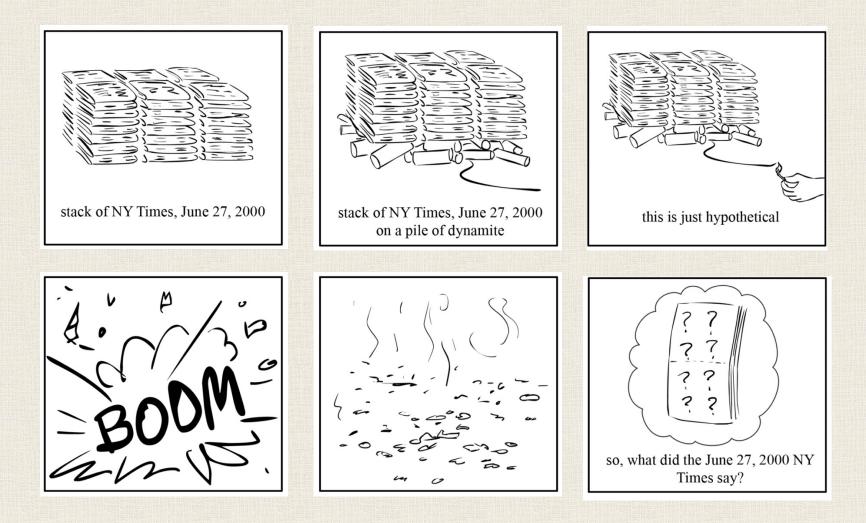


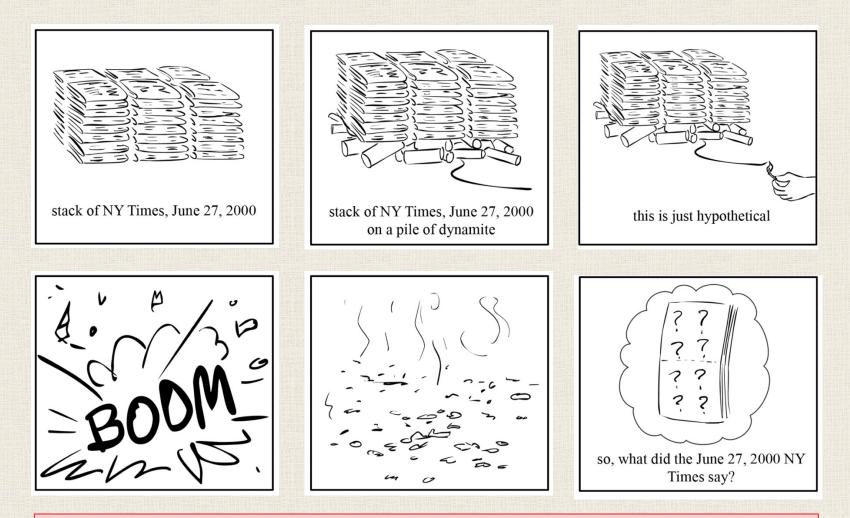
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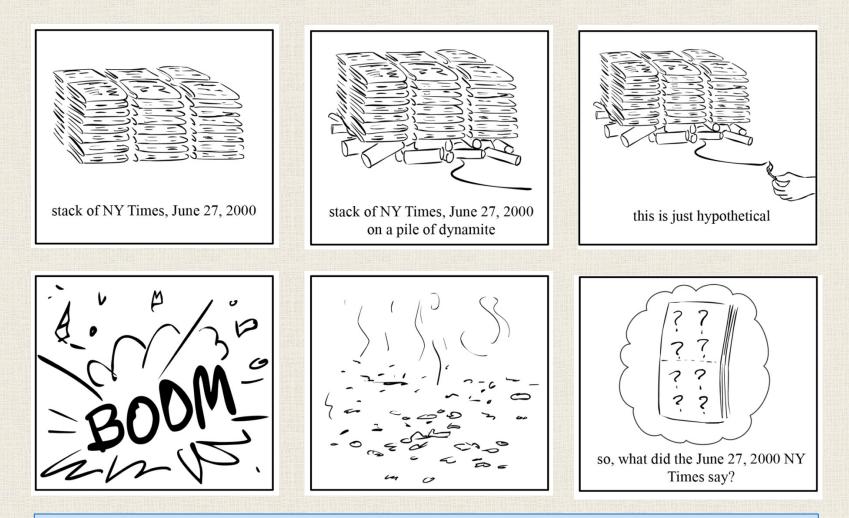




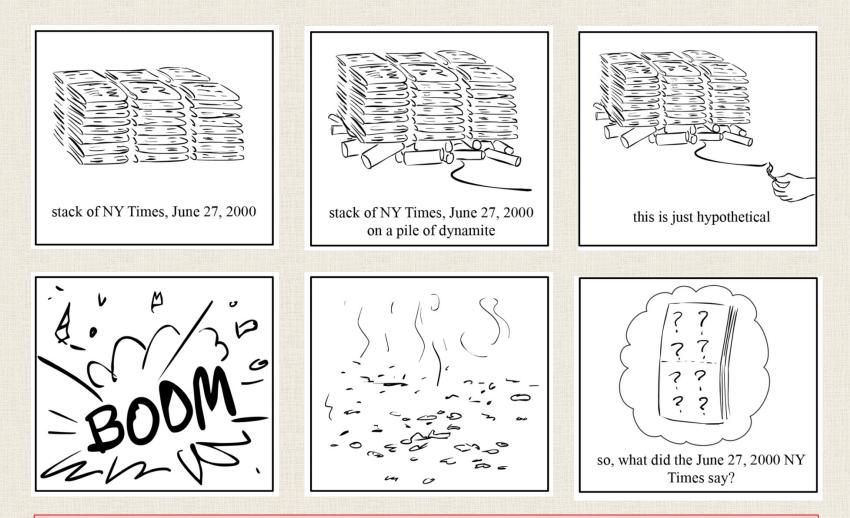




STOP: How would you reconstruct the news?

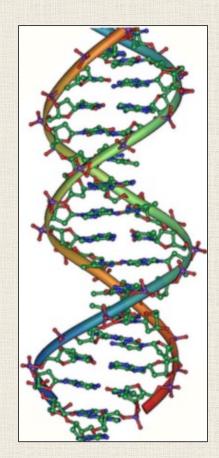


The Newspaper Problem is an overlap puzzle.

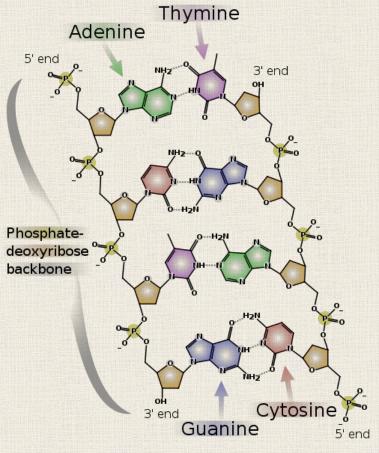


But what does this have to do with biology?

DNA is a Double Helix of Nucleotide Strands



DNA's Double Helix (1953)



DNA's Molecular Structure

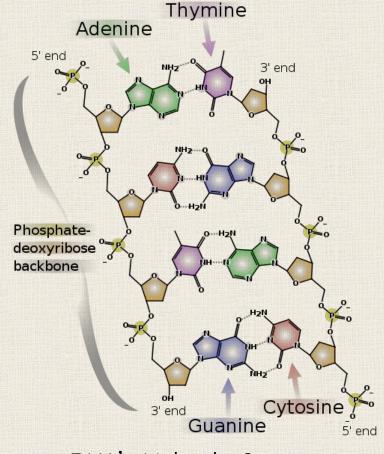
Courtesy: Madprime, Wikimedia Commons

The Order of Nucleotides Determines Genetics

Nucleotide: Half of one "rung" of DNA.

Four choices for the nucleic acid of a nucleotide:

- 1. Adenine (A)
- 2. Cytosine (C)
- 3. Guanine (G)—bonds to C
- 4. Thymine (T)—bonds to A



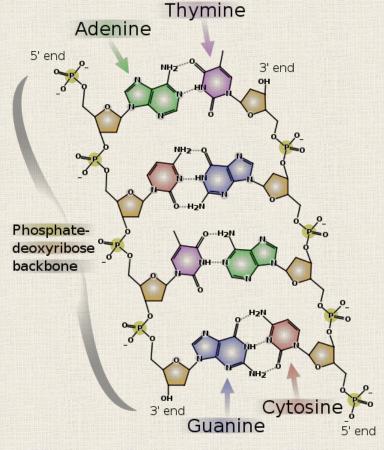
DNA's Molecular Structure

Courtesy: Madprime, Wikimedia Commons

The Order of Nucleotides Determines Genetics

Nucleotide: Half of one "rung" of DNA.

Key point: if we know one strand of DNA, we get the other strand for free because of this "complementarity".



DNA's Molecular Structure

Courtesy: Madprime, Wikimedia Commons

Genome "Sequencing" Means "Reading" the Genome

Genome: The nucleotide sequence read down one side of an organism's chromosomal DNA. A human genome has about 3 billion letters.

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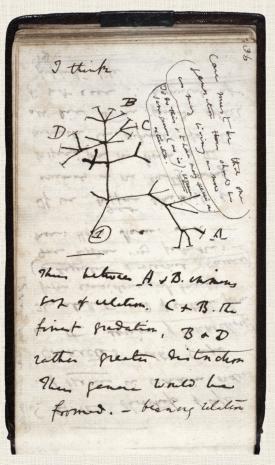
Polychaos dubium (an amoeba) has one of the longest known genomes: 670 billion nucleotides.

Genome "Sequencing" Means "Reading" the Genome

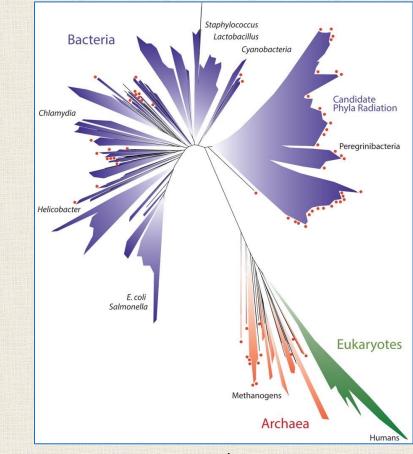
Genome: The nucleotide sequence read down one side of an organism's chromosomal DNA. A human genome has about 3 billion letters.

Key Point: DNA is submicroscopic! How do we read something that we cannot see?

We Sequence a Species's Genome to Unlock its Genetic Identity



Darwin's notebook c. 1837



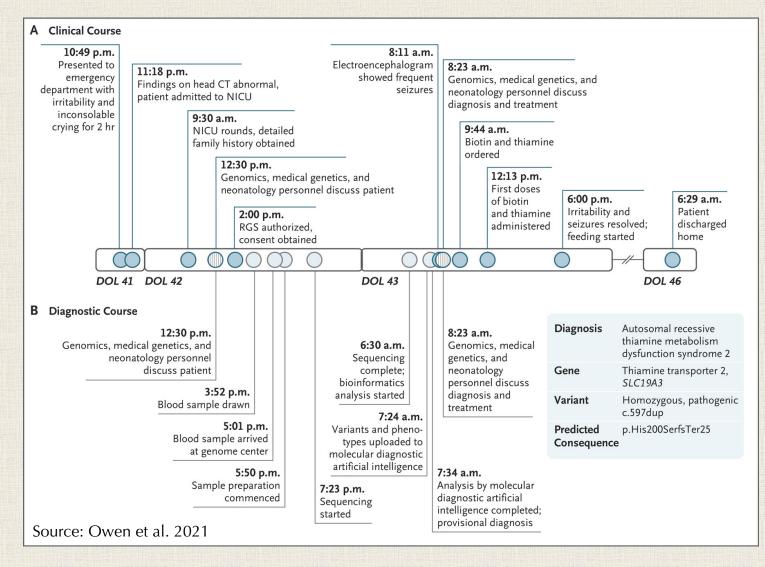
Hug et al., 2016 Nature Biotechnology, Discovery Magazine

We Sequence an Individual's Genome to Find What Makes them Unique

2011: First person whose life was saved because of genome sequencing.



Ten years later, genome sequencing saves a life in 13 hours

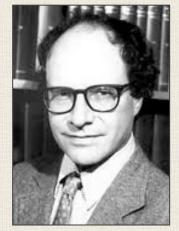


History of Genome Sequencing

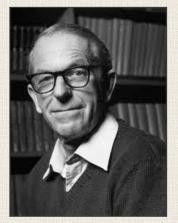
Late 1970s: Walter Gilbert and Frederick Sanger develop independent sequencing methods.

GAGTTTTATCGCTTCCATGACGACAGATTAACACTTTCGGATATTTCTGATGAGGCGAAAAATTATCTTGATAAAGCAGGAATTACTACTGCTTGTTTACGAATGAGAAGTGGACGGCGGAAAATGAGAAAATTCGACCTATCCTTGCGCAGCT GCTCGTTATGGTTTCCGTTGCTGCCATCTCAAAAACATTTGGACTGCTCCGCCTTCCTCCGGGACTGAGCTTTCTCGCCAAATGACGACTTCTACCACATTTGGCATACTGGCTCTGCCAAGCTGCTAATTTGGACTGACCAA TCAACAGACCTATAAACATTCTGTGCCGCGCTTTCTTTGTTCCTGAGCATGGCACTATGTTTGCTGCGCTTGTTCGTTTTCCGCCTACTGCGACTAAAGAGATTCAGTACCCTAAAGGTGCTTTGACCTTATACCGATATTGCTGGCGACCCTGTT TTGTATGGCAACTTGCCGCCGCGGGAAATTTCTATGAAGGATGTTTTCCGTTCTGGTGATCGTCTAAGAAGTTTAAGATGCTGAGGGTCAGTGGTATCGTTATG AACCGCCTTCTCGCTGATTGCAAGAACGCCGTACTTATTCCCCACCATGATTGCAGCCCGCTTCCAGTTGCAGTGCAGTGGAATAGTCAGGTTAAAATTAAATGTGA TGGTTCTCACTTCTGTTACTCCAGCTTCTTCGGCACCTGTTTTACAGACACCTAAAGCTACATCGTCAACGTTATATTTTGATAGTTTGACGGTTAATGCTGGTAATGGTGGTTTTCTTC TGGTGCTATTGCTGGCGGTATTGCTTCTGCTCTTGCTGGTGGCGCCCATGTCTAAATTGTTTGGAGGCGGTCAAAAAGCCGCCTCCGGTGGCATTCAAGGTGATGTGCTTCCCGATAACAATACTGTAG CCTGCA

Bacterial phage PhiX174 genome (5,386 nucleotides)



Walter Gilbert



Frederick Sanger

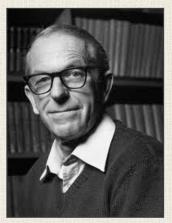
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1980: They share the Nobel Prize in Chemistry.



Walter Gilbert



Frederick Sanger

History of Genome Sequencing

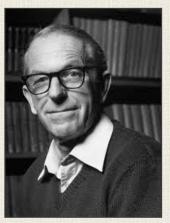
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1980: They share the Nobel Prize in Chemistry.

However, their approaches cost about \$1 per nucleotide.

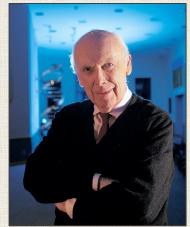


Walter Gilbert



Frederick Sanger

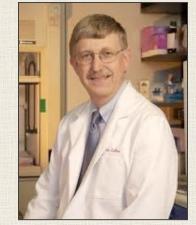
1990: Human Genome Project given \$3 billion to sequence human genome.



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1992: James Watson resigns, replaced by Francis Collins.

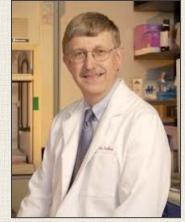


Francis Collins

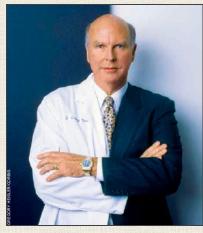
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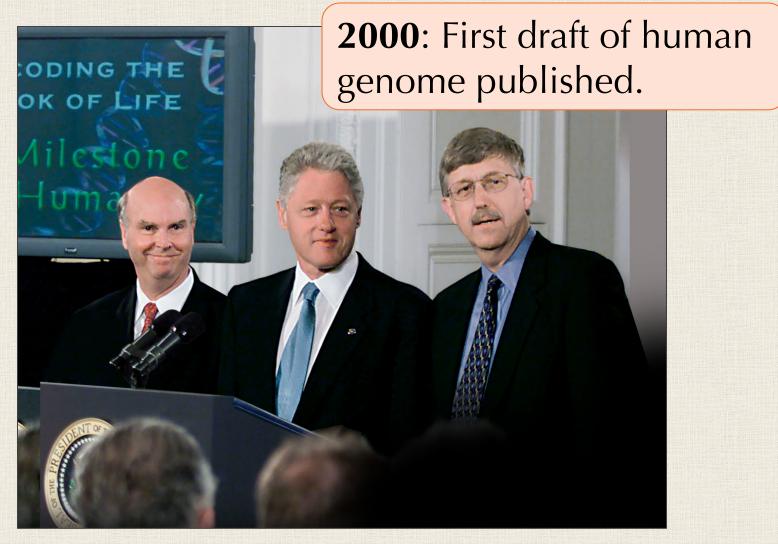
1997: Craig Venter founds Celera Genomics with same goal.



Francis Collins

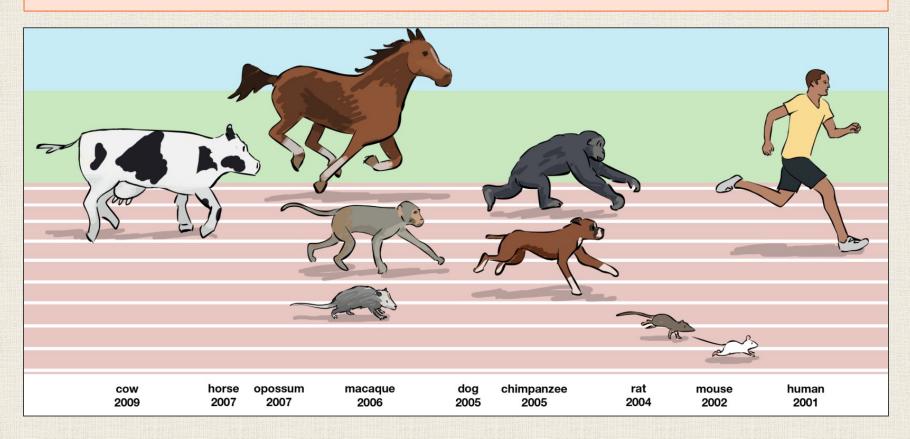


Craig Venter



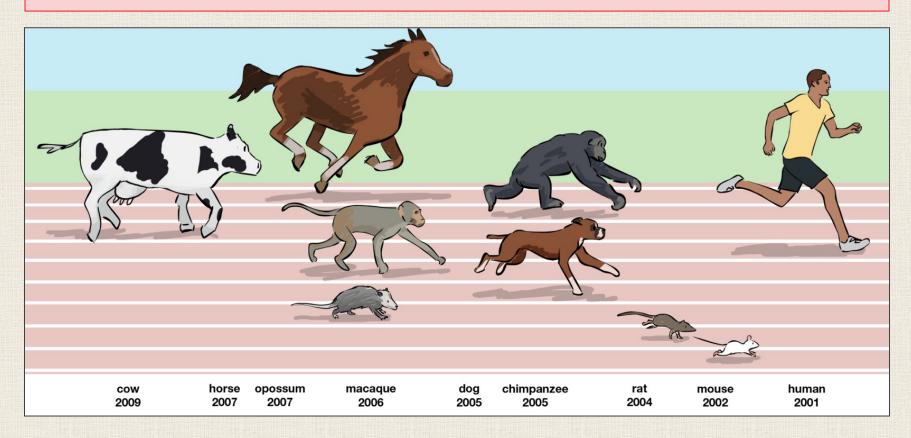
From One Mammal Genome to Many

Early 2000s: Many more mammalian genomes are sequenced using Sanger's approach.

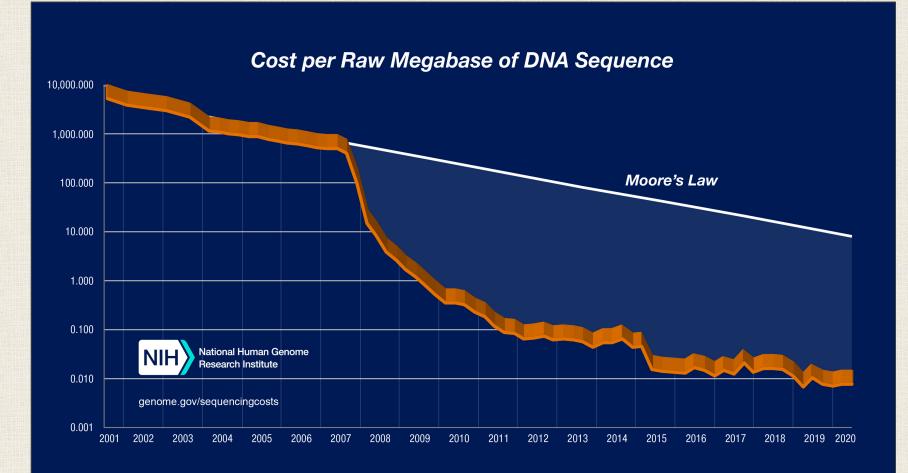


From One Mammal Genome to Many

Problem: This approach was just too expensive to scale to thousands of species.



Sequencing Cost Has Fallen Faster than Moore's Law



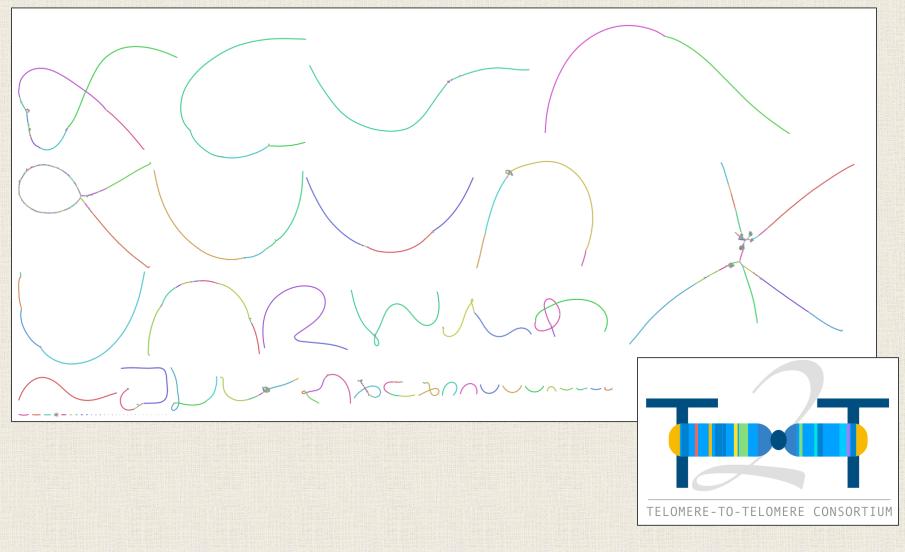
GISAID collects 400k 2 Million SARS-CoV-2 Genomes in One Year-Two Years



Scientists aim to sequence 1.5M eukaryotes before 2030

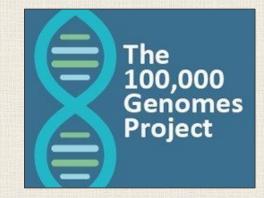
EARTH BIOGENOME PROJECT

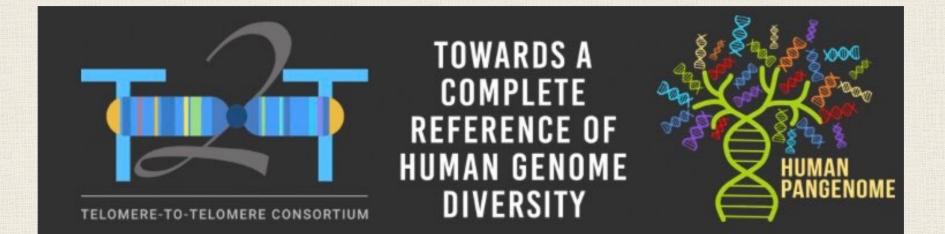
Dark Secret: The First *Full* Human Genome Wasn't Sequenced Until 2020!



We Now Have Over 2 Million Human Genomes

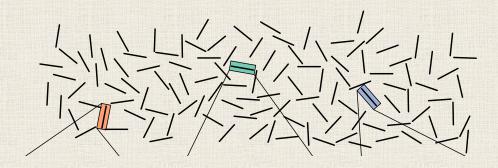
100,000 Genomes: Sequenced 100,000 UK resident genomes (2012-2018).





Multiple identical copies of a genome

Multiple identical copies of a genome

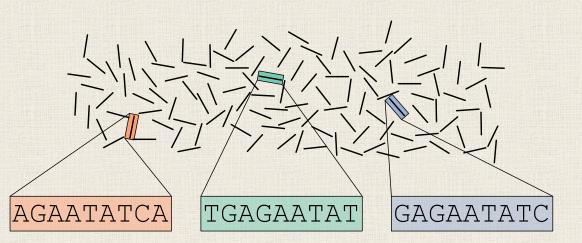


Shatter the genome into reads

Multiple identical copies of a genome

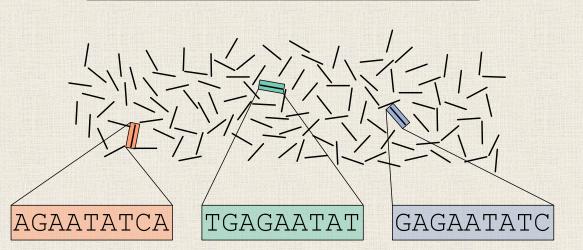
Shatter the genome into reads

Sequence the reads (Lab)



Multiple identical copies of a genome

Shatter the genome into reads



Sequence the reads (Lab)

Assemble the genome using overlapping reads (Computational) AGAATATCA GAGAATATC TGAGAATAT . TGAGAATATCA...

Multiple identical copies of a genome

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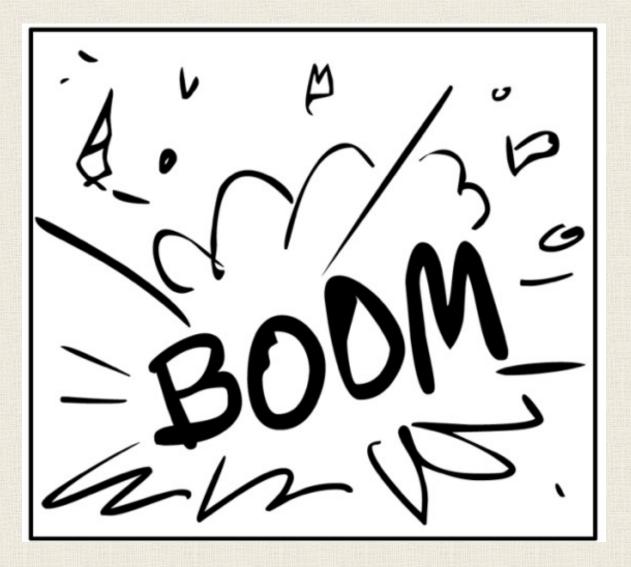
TGAGAATAT

GAGAATATC

What does genome sequencing remind you of?

AGAATATCA

Genome Assembly = Overlap Puzzle



Interlude: How Are Reads Sequenced?



https://www.youtube.com/watch?v=fCd6B5HRaZ8

A COMPUTATIONAL PROBLEM FOR GENOME ASSEMBLY

Practical Sequencing Complications

1. DNA may be divided over **multiple chromosomes**.

2. Reads have **imperfect "coverage"** of the underlying genome – there may be some regions that are not covered by any reads.

3. Sequencing machines are error-prone.

4. DNA is **double-stranded**.

Making Some Assumptions is OK!

1. A genome consists of a single chromosome.

2. Reads have **perfect "coverage"** of the underlying genome –every possible starting position gets sampled by the sequencer.

3. Sequencing machines are error-free.

4. DNA is **single-stranded**.

Genome Assembly Problem

- Input: A collection of strings *Reads*.
- **Output:** A string *Genome* reconstructed from *Reads*.

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STOP: Is this a well-defined problem?

Genome Assembly Problem

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- **Output:** A string *Genome* reconstructed from *Reads*.

STOP: Is this a well-defined problem?

Answer: No! We have no sense of what it means to "reconstruct" a genome.

The *k*-mer composition of a string *Text*, denoted *Composition_k*(*Text*), is the collection of all *k*-mer substrings of *Text* (including repeats).

NANABANANA NAN	3-mer composition
ANA	
NAB	
ABA	
BAN	
ANA	
NAN	
ANA	

We want to solve the *reverse* problem: given a collection of strings, find a string having this collection as its *k*-mer composition.

String Reconstruction Problem

- **Input:** A collection of strings *patterns* and an integer *k*.
- **Output:** A string *Text* whose *k*-mer composition is equal to *Patterns*.

STOP: Now is this a well-defined computational problem?

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Answer: Not quite ... what if *Patterns* = {AAA, ZZZ}?

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String Reconstruction Problem

- **Input:** A collection of strings *patterns* and an integer *k*.
- **Output:** A string *Text* whose *k*-mer composition is equal to *Patterns* (if such a string exists).

SOLVING THE STRING RECONSTRUCTION PROBLEM?

Exercise: Reconstruct the string corresponding to the following 3-mer composition.

AAT ATG GTT TAA TGT

Exercise: Reconstruct the string corresponding to the following 3-mer composition.

AAT	ATG	GTT	TAA	TGT
-----	-----	-----	-----	-----

TAA AAT ATG TGT GTT TAATGTT

"Greedy" algorithm: for each *k*-mer, look for the *k*-mer of maximum overlap in each direction.

> TAA AAT ATG TGT GTT TAATGTT

"Greedy" algorithm: for each *k*-mer, look for the *k*-mer of maximum overlap in each direction.

Genome assembly is trivial! We can pack up and go home.

"Greedy" algorithm: for each k-mer, look for the k-mer of maximum overlap in each direction.

Genome assembly is trivial! We can pack up and go home.

Exercise: Apply this algorithm to the 3-mer composition at right.

AAT ATG ATG ATG CAT CCA GAT GCC GGA GGG GTT TAA TGC TGG TGT

© 2024 Phillip Compeau

AAT ATG ATG

ATG CAT CCA GAT

GCC GGA GGG

GGG GTT TAA TGC TGG

TGT

TAA				AAT
				ATG
				ATG
				ATG
				CAT
				CCA
				GAT
				GCC
				GGA
				GGG
				GTT
				TAA
				TGC
				TGG
TAA				TGT

TAA			AAT
AAT			ATG
			ATG
			ATG
			CAT
			CCA
			GAT
			GCC
			GGA
			GGG
			GTT
			TAA
			TGC
			TGG
TAAT			TGT

TAA		AAT
AAT		ATG
ATG		ATG
		ATG
		CAT
		CCA
		GAT
		GCC
		GGA
		GGG
	STOP: Which one	GTT
		TAA
	should we choose?	TGC
		TGG
TAATG		TGT

TAA				AAT	C
AAT				ATC	Ę
ATG				ATG	-
TGC				ATG	ק
				CAT	[
				CCA	7
				GAT	C
				GCC	2
				GGA	Ł
				GGG	r T
				GTI	C
				TAA	Ŧ
				TGC	7
				TGG	Ę

TGT

TAATGC

TAA	AAT
AAT	ATG
ATG	ATG
TGC	ATG
GCC	CAT
	CCA
	GAT
	GCC
	GGA
	GGG
	GTT
	TAA
	TGC
	TGG
TAATGCC	TGT

TAA	AAT
AAT	ATG
ATG	ATG
TGC	ATG
GCC	CAT
CCA	CCA
	GAT
	GCC
	GGA
	GGG
	GTT
	TAA
	TGC
	TGG
TAATGCCA	TGT

TAA	AAT
AAT	ATG
ATG	ATG
TGC	ATG
GCC	CAT
CCA	CCA
CAT	GAT
	GCC
	GGA
	GGG
	GTT
	TAA
	TGC
	TGG
TAATGCCAT	TGT

TAA	AAT
AAT	ATG
ATG	ATG
TGC	ATG
GCC	CAT
CCA	CCA
CAT	GAT
ATG	GCC
	GGA
	GGG
	GTT
	TAA
	TGC
	TGG
TAATGCCATG	TGT

TAATGCCATG

TAA	AAT
AAT	ATG
ATG	ATG
TGC	ATG
GCC	CAT
CCA	CCA
CAT	GAT
ATG	GCC
TGG	GGA
	GGG
	GTT
	TAA

TGC

TGG

TGT

TAATGCCATGG

TAA	AAT
AAT	ATG
ATG	ATG
TGC	ATG
GCC	CAT
CCA	CCA
CAT	GAT
ATG	GCC
TGG	GGA
GGA	GGG
	GTT
	TAA
	TGC
	TGG

TGT

TAATGCCATGGA

TAA	AAT
AAT	ATG
ATG	ATG
TGC	ATG
GCC	CAT
CCA	CCA
CAT	GAT
ATG	GCC
TGG	GGA
GGA	GGG
GAT	GTT
	TAA
	TGC
	TGG

TGT

TAATGCCATGGAT

TAA	AAT
AAT	ATG
ATG	ATG
TGC	ATG
GCC	CAT
CCA	CCA
CAT	GAT
ATG	GCC
TGG	GGA
GGA	GGG
GAT	GTT
ATG	TAA
	TGC
	TGG
TAATGCCATGGATG	TGT

TAA	AAT
AAT	ATG
ATG	ATG
TGC	ATG
GCC	CAT
CCA	CCA
CAT	GAT
ATG	GCC
TGG	GGA
GGA	GGG
GAT	GTT
ATG	TAA
TGT	TGC
	TGG
TAATGCCATGGATGT	TGT

TAA		AAT
AAT		ATG
ATG		ATG
TGC		ATG
GCC		CAT
CCA		CCA
CAT		GAT
ATG		GCC
TGG		GGA
GGA	???	GGG
GAT		GTT
ATG		TAA
TGT		TGC
GTT		TGG
TAATGCCATGGATGTT		TGT

TAA			AAT
AAT			ATG
ATG			ATG
TGC	STOP: Why did our		ATG
GCC	algorithm fail?		CAT
CCA			CCA
CAT			GAT
ATG	5		GCC
TG	G		GGA
G	GA	;;;	GGG
GAT		GTT	
ATG		TAA	
	TGT		TGC
	GTT		TGG
TAATGCCATG	GATGTT		TGT

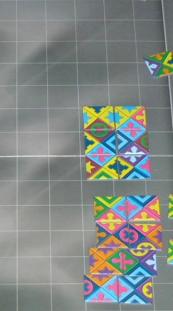
TAA			AAT
AAT			ATG
ATG			ATG
TGC	Answer: Repeated		ATG
GCC	substrings!		CAT
CCA			CCA
CAT			GAT
ATG	5		GCC
TG	G		GGA
G	GA	???	GGG
	GAT		GTT
ATG			TAA
	TGT		TGC
	GTT		TGG
TAATGCCATG	GATGTT		TGT



ENER

MONTH FURNING



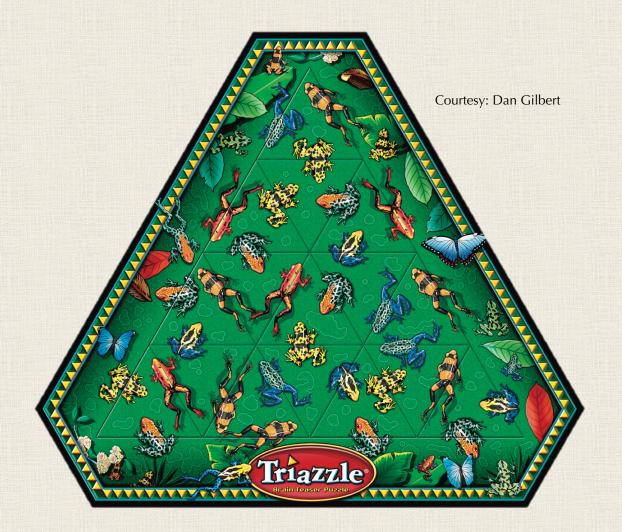


X

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Courtesy: Matej Bat'ha

... Even a 16-piece "Triazzle" Can Take a Human Hours to Solve...



... and Repeats Complicate Genome Assembly Too 🐵

Repeats are very common in genomes; the 300nucleotide **Alu repeat** occurs over a million times (with minor changes) in every human genome.

... and Repeats Complicate Genome Assembly Too 🐵

Repeats are very common in genomes; the 300nucleotide **Alu repeat** occurs over a million times (with minor changes) in every human genome.

So what hope do we have of assembling a genome?

GENOME ASSEMBLY AS A HAMILTONIAN PATH PROBLEM

Solution to Previous Exercise

STOP: Is this the only solution?

TAA	AAT
AAT	ATG
ATG	ATG
TGC	ATG
GCC	CAT
CCA	CCA
CAT	GAT
ATG	GCC
TGG	GGA
GGG	GGG
GGA	GTT
GAT	TAA
ATG	TGC
TGT	TGG
GTT	TGT
TAATGCCATGGGATGTT	

Genome path: assign each read to a node, connect adjacent reads with edges.



Genome path: assign each read to a node, connect adjacent reads with edges.

 \rightarrow (CAT) \rightarrow (ATG) \rightarrow (TGG)

→(GGG)

→GGA

GAT

STOP: Can you still see the genome?

->(CCA)-

→(GCC)

>(TGC)

Genome path: assign each read to a node, connect adjacent reads with edges.

STOP: Can you still see the genome?

→(CCA)-

>(GCC)

STOP: Could you construct the genome path if you only knew the 3-mer composition?

 \rightarrow (CAT) \rightarrow (ATG) \rightarrow (TGG) \rightarrow (GGG)

GAT

→GGA

Genome path: assign each read to a node, connect adjacent reads with edges.

STOP: Can you still see the genome?

->(CCA)-

>(TGC)

>(GCC)

Answer: No ... we need to know the order of the *k*-mers.

→CAT)→ATG)→(TGG)-

→(GGG)

→GGA

>(GAT)

- **Prefix:** First k 1 letters in a k-mer.
- **Suffix:** Last *k* 1 letters in a *k*-mer.

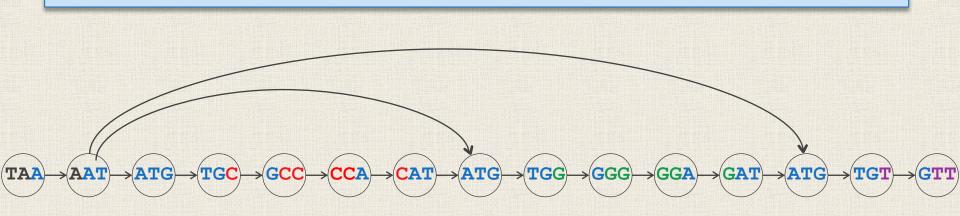


- **Prefix:** First *k* 1 letters in a *k*-mer.
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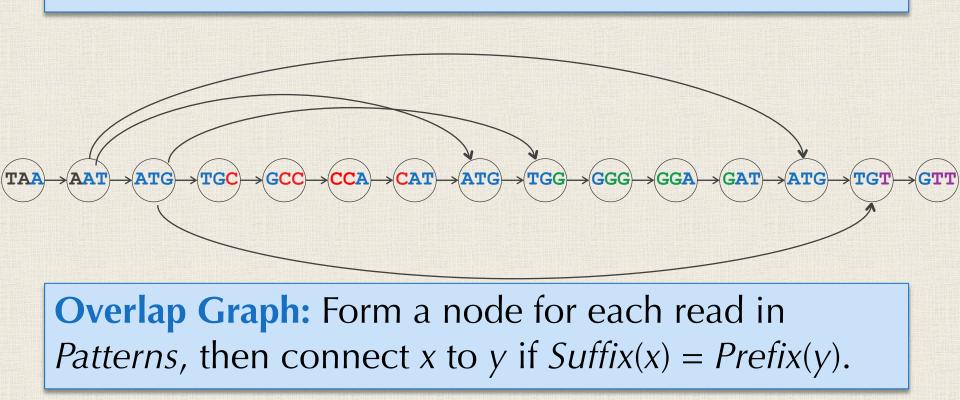
Overlap Graph: Form a node for each read in *Patterns,* then connect x to y if Suffix(x) = Prefix(y).

- **Prefix:** First *k* 1 letters in a *k*-mer.
- **Suffix:** Last *k* 1 letters in a *k*-mer.

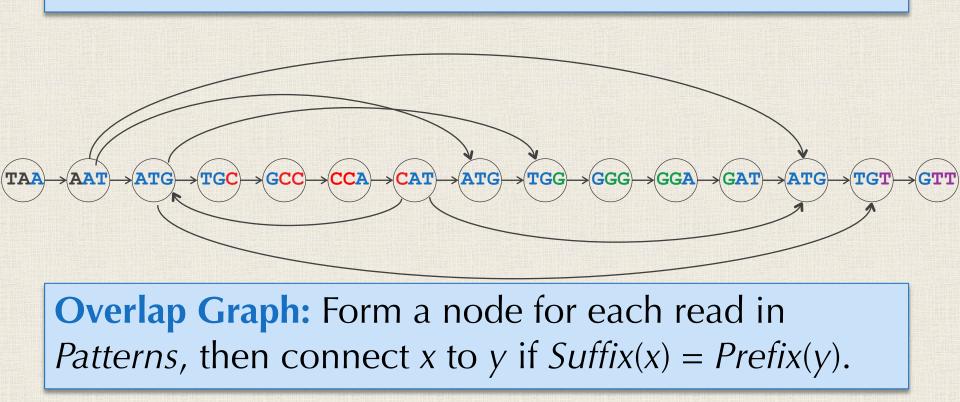


Overlap Graph: Form a node for each read in *Patterns,* then connect *x* to *y* if Suffix(x) = Prefix(y).

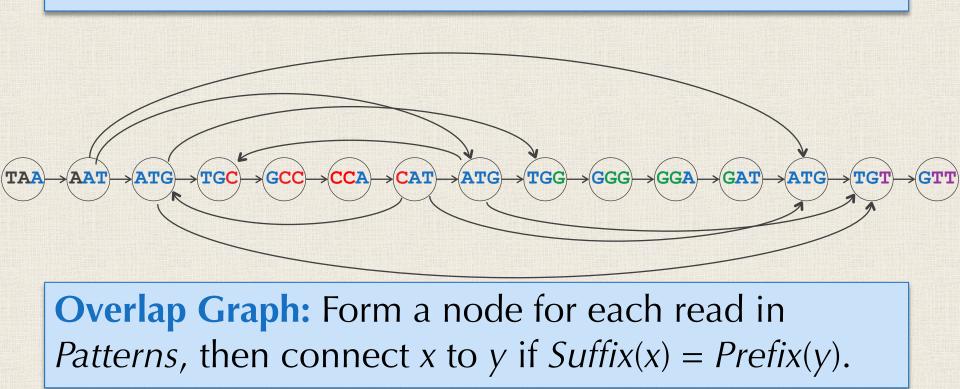
- **Prefix:** First *k* 1 letters in a *k*-mer.
- **Suffix:** Last *k* 1 letters in a *k*-mer.



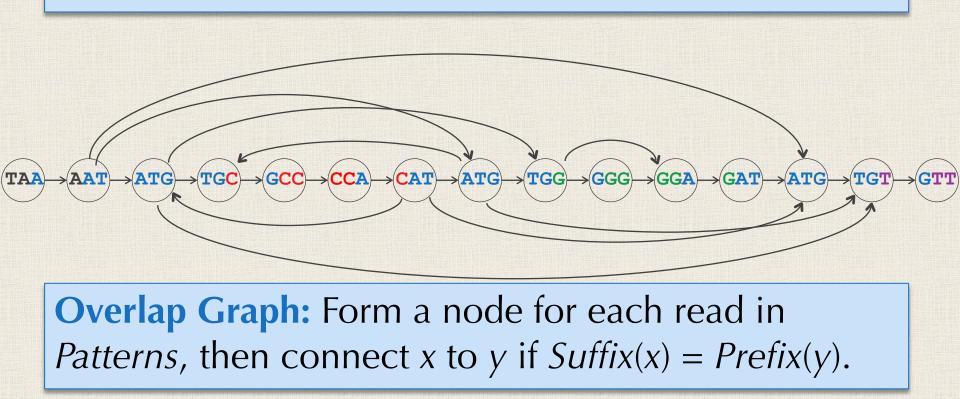
- **Prefix:** First *k* 1 letters in a *k*-mer.
- **Suffix:** Last *k* 1 letters in a *k*-mer.



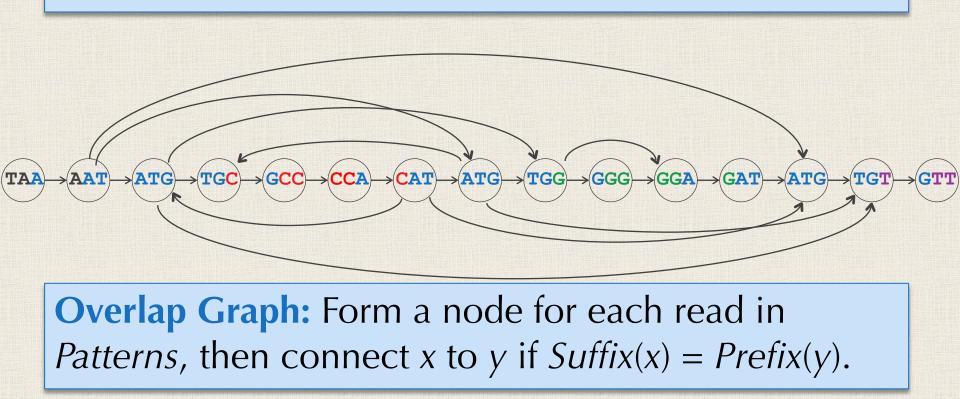
- **Prefix:** First *k* 1 letters in a *k*-mer.
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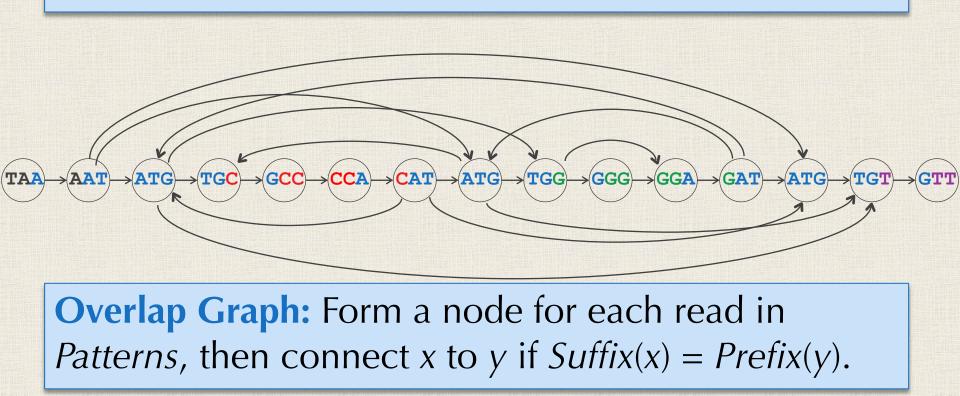
- **Prefix:** First *k* 1 letters in a *k*-mer.
- **Suffix:** Last *k* 1 letters in a *k*-mer.



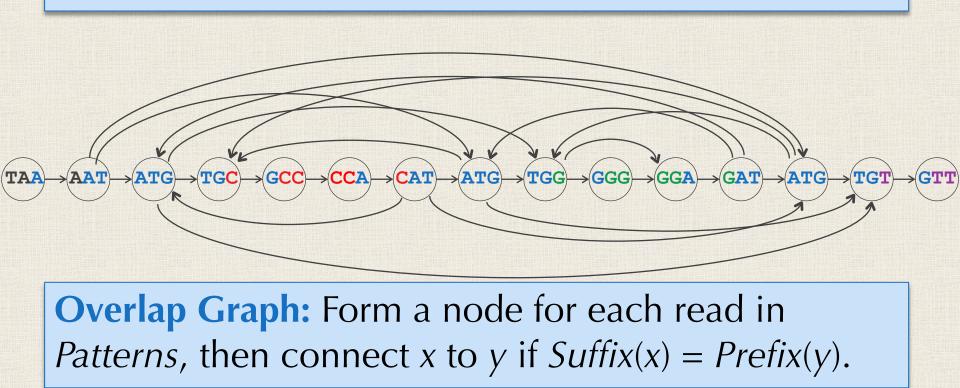
- **Prefix:** First *k* 1 letters in a *k*-mer.
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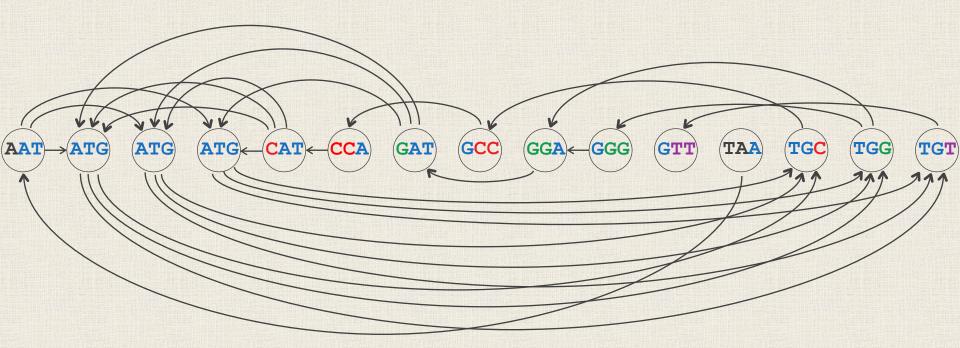
- **Prefix:** First *k* 1 letters in a *k*-mer.
- **Suffix:** Last *k* 1 letters in a *k*-mer.



Note: we can still see the genome path, but we wouldn't if we don't know the order of *k*-mers ...

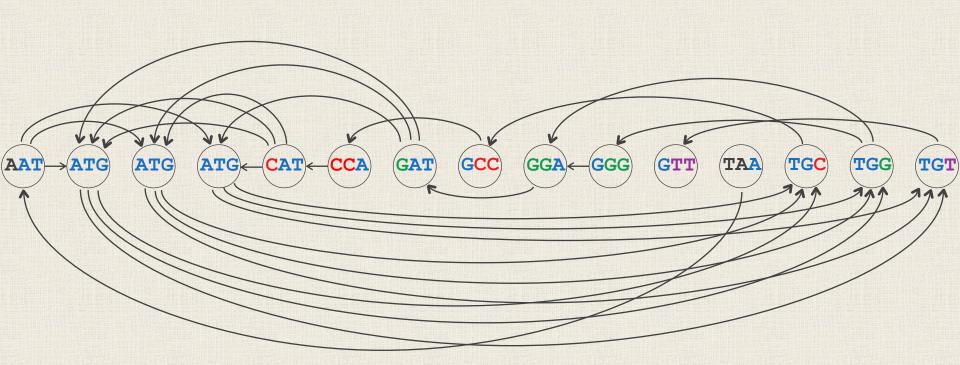
CAT ATG TGG **Overlap Graph:** Form a node for each read in *Patterns,* then connect x to y if Suffix(x) = Prefix(y).

Arranging k-mers Lexicographically Makes Genome Vanish



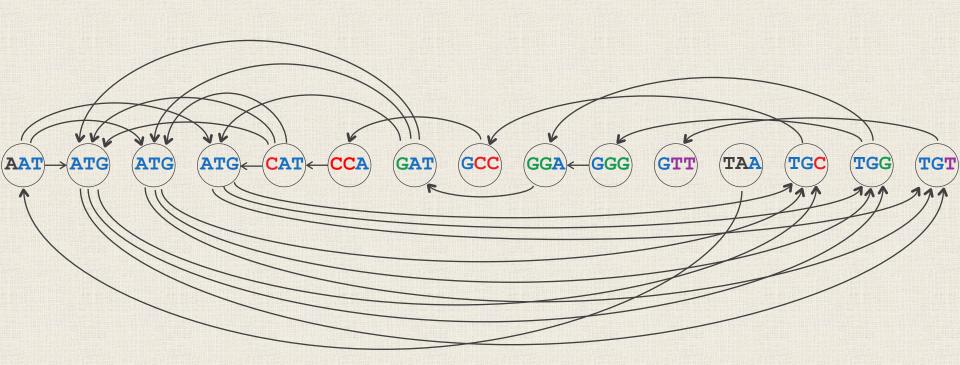
Arranging k-mers Lexicographically Makes Genome Vanish

STOP: If we gave you this graph, what would you look for to find the genome?



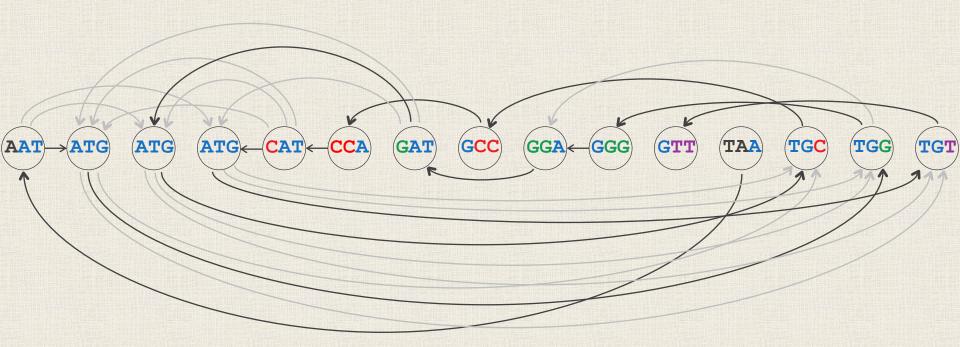
We are Looking for a Hamiltonian Path in the Overlap Graph

Hamiltonian path: A path through a graph that touches each node exactly once.



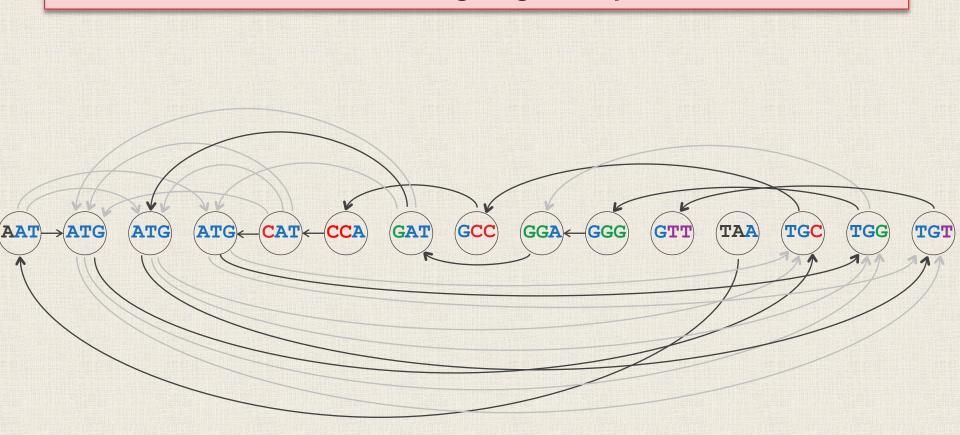
Here's One Solution

STOP: What genome does the highlighted path reconstruct?



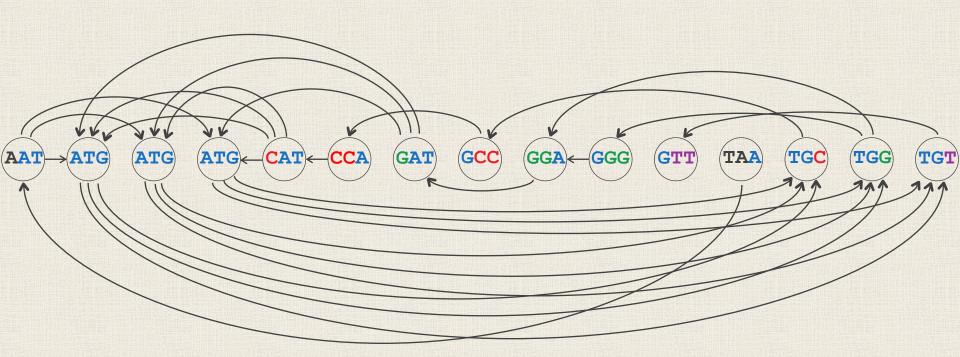
And Here's Another Solution

STOP: How about this highlighted path?



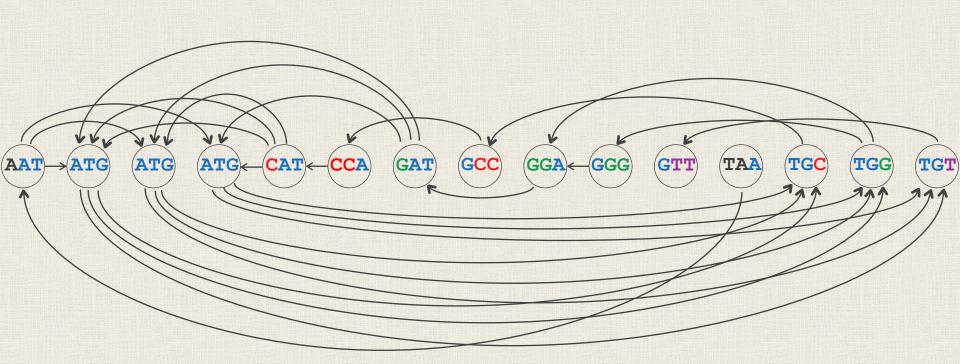
We are Looking for a Hamiltonian Path in the Overlap Graph

Note: The graph organizes our reads, but we don't have an *algorithm* for finding a Hamiltonian path.



We are Looking for a Hamiltonian Path in the Overlap Graph

STOP: What does the overlap graph look like if there are many repeats? What if there are none?



Aside 1: de Bruijn and Good

A binary string is *k*-universal if it contains every binary *k*-mer once.

Exercise: Find a 3-universal string.



Jack Good

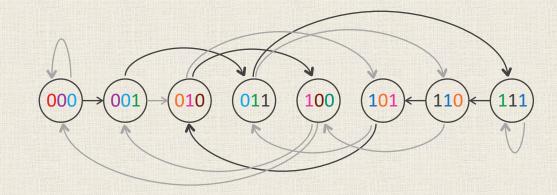


Nicolaas de Bruijn

Aside 1: de Bruijn and Good

A binary string is *k*-universal if it contains every binary *k*-mer once.

Note: a *k*-universal string corresponds to a Hamiltonian path in the following overlap graph.





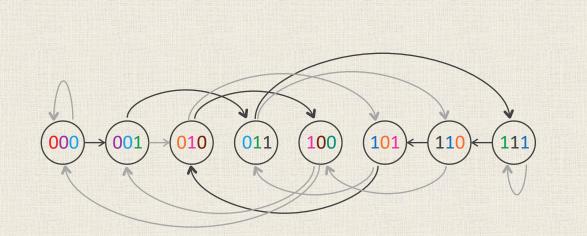
Jack Good



Nicolaas de Bruijn

Aside 1: de Bruijn and Good

1946: Good and de Bruijn independently discover a way to find *k*-universal strings. They cannot imagine that their approach will one day power genome sequencing.



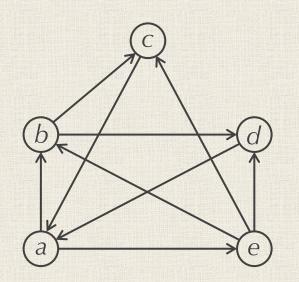


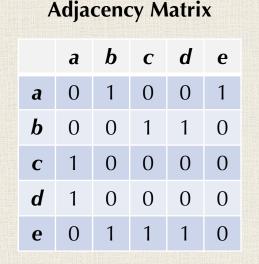
Jack Good



Nicolaas de Bruijn

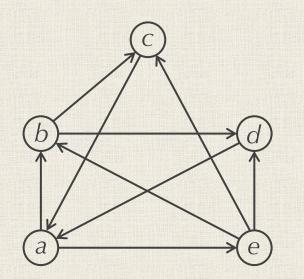
Aside 2: Two Ways to Represent Graphs Computationally





Adjacency matrix: $A_{i,j} = 1$ if there is an edge connecting node *i* to node *j*; $A_{i,j} = 0$ otherwise.

Aside 2: Two Ways to Represent Graphs Computationally



Adjacency Matrix							Adjacency List	
		а	b	С	d	е		
	а	0	1	0	0	1	а	<i>b,</i> e
	b	0	0	1	1	0	b	<i>c, d</i>
	С	1	0	0	0	0	С	а
	d	1	0	0	0	0	d	а
	e	0	1	1	1	0	e	b, c, d

Adjacency matrix: $A_{i,j} = 1$ if there is an edge connecting node *i* to node *j*; $A_{i,j} = 0$ otherwise.

Adjacency list: Dictionary; "key" node *i*; "value" is list of nodes that *i* is connected to.

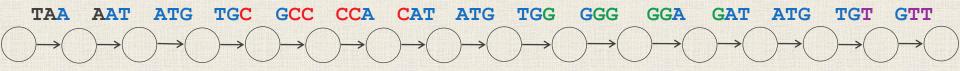
GENOME ASSEMBLY AS AN EULERIAN PATH PROBLEM

We start again with a "genome path" corresponding to TAATGCCATGGGATGTT.

 TAA
 AAT
 ATG
 TGC
 GCC
 CCA
 CAT
 ATG
 TGG
 GGG
 GGA
 GAT
 ATG
 TGT
 GTT

 \rightarrow \rightarrow

We start again with a "genome path" corresponding to TAATGCCATGGGATGTT.

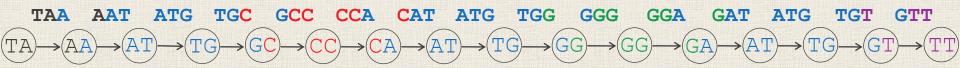


STOP: How should we label the nodes?

Each node represents the (k - 1)-mer corresponding to the *overlap* between adjacent edges.

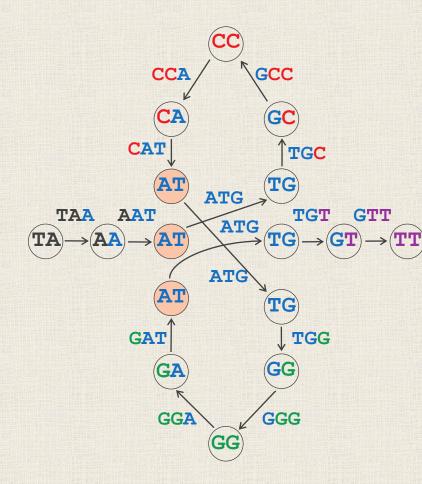


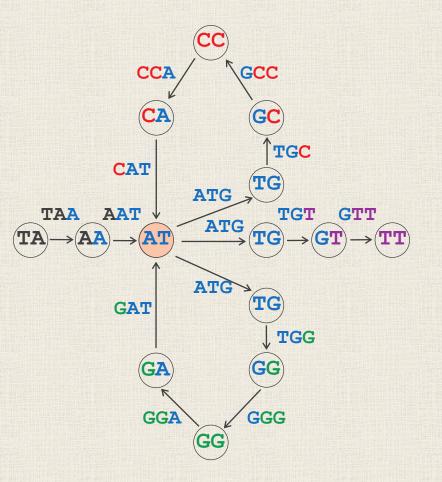
Each node represents the (k - 1)-mer corresponding to the *overlap* between adjacent edges.



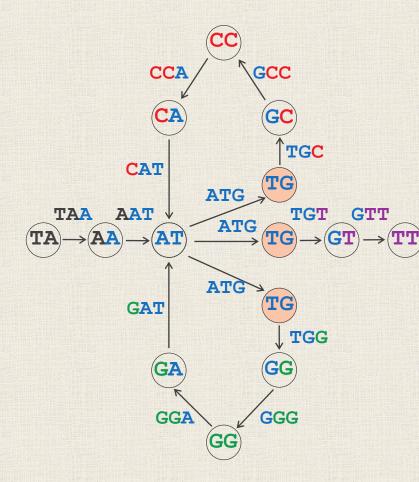
Unlike with the overlap graph, we will *glue* together nodes that have the same label.

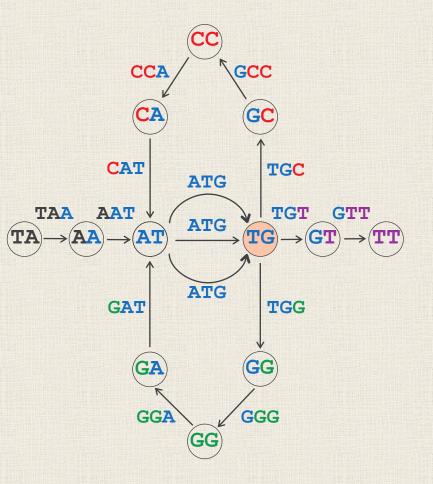
First: Gluing AT Together

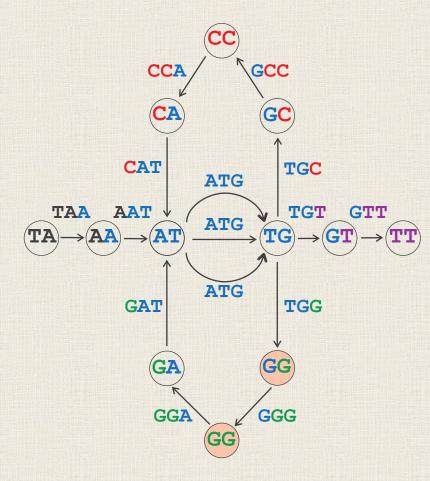


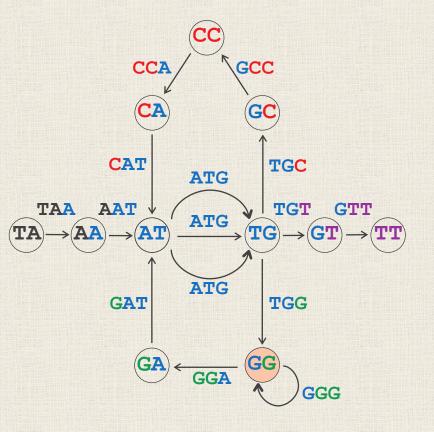


Next: Gluing TG Together

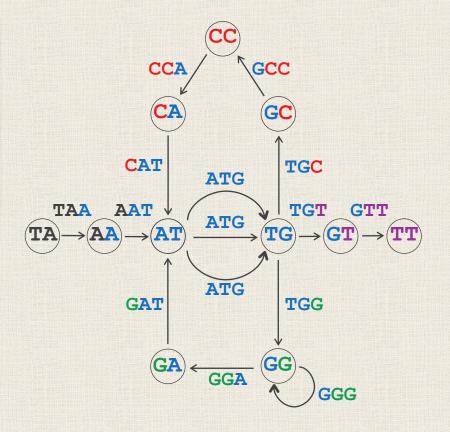






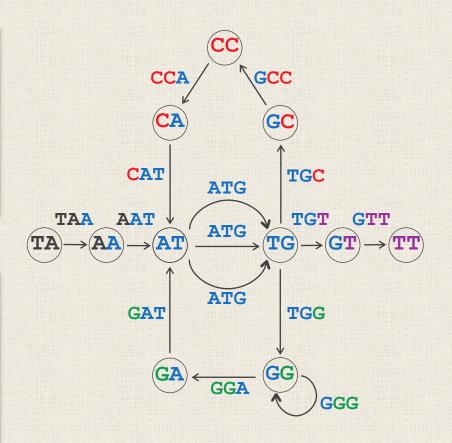


This graph is called the **de Bruijn graph** of *Text* = TAATGCCATGGGATGTT for k = 3.



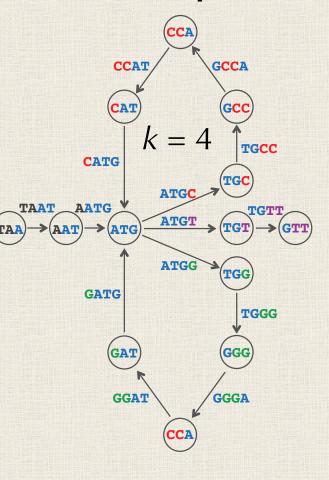
This graph is called the **de Bruijn graph** of *Text* = TAATGCCATGGGATGTT for k = 3.

Exercise: Construct the de Bruijn graphs for k = 4 and k = 5. How do they differ from k = 3?



de Bruijn Graph Becomes Less "Tangled" as *k* Increases (fewer repeats)

k = 3CCA GCC CAT TGC ATG AAT TGT TAA GTT ATG ATG GAT TGG GA GGA GGG

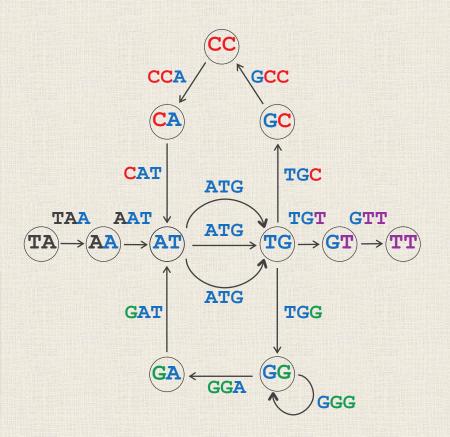


k = 5



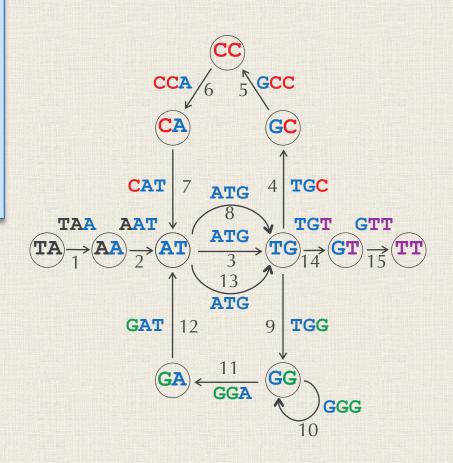
This graph is called the **de Bruijn graph** of *Text* = TAATGCCATGGGATGTT for k = 3.

STOP: If we gave you this graph, could you reconstruct *Text*? How?



The Genome Path is Still There

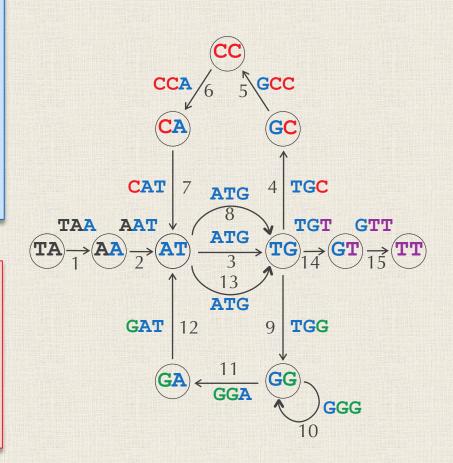
The genome path is an **Eulerian path** in the de Bruijn graph, or a path that uses every edge exactly once.



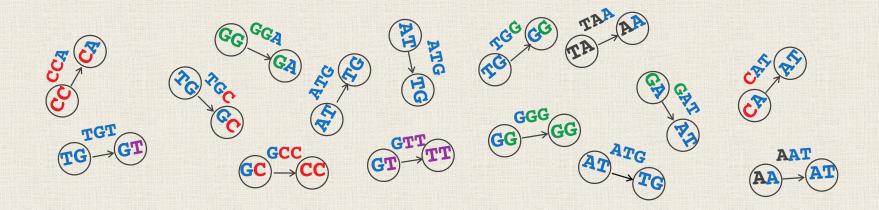
The Genome Path is Still There

The genome path is an **Eulerian path** in the de Bruijn graph, or a path that uses every edge exactly once.

STOP: Can you construct the de Bruijn graph if you don't already know *Text*?

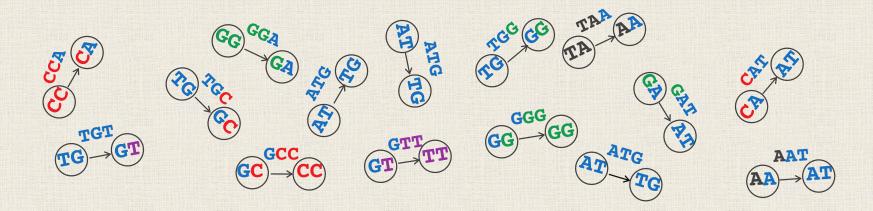


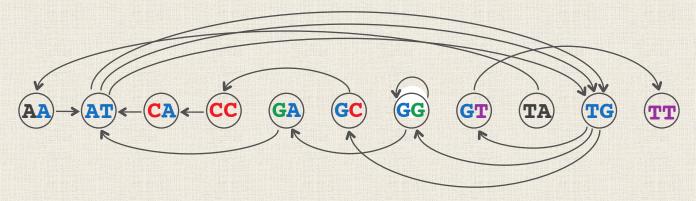
Forming de Bruijn Graph from k-mers



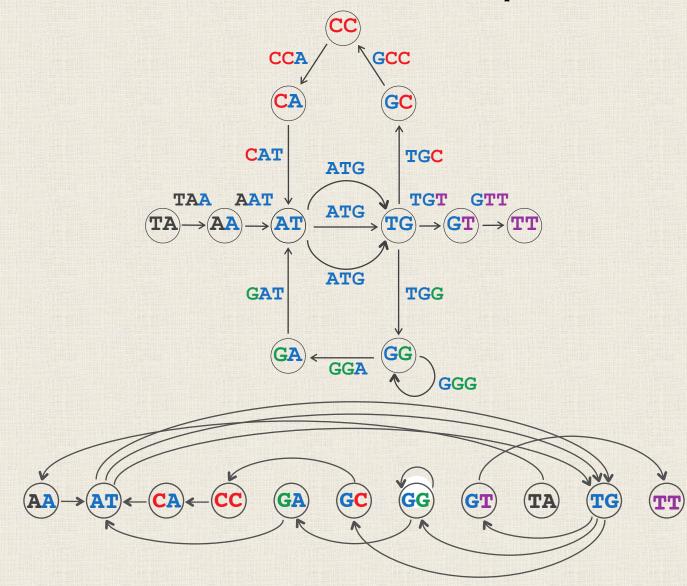
Exercise: Here are the 3-mers from our original dataset represented as *isolated edges*. By gluing nodes together, what do you obtain?

Forming de Bruijn Graph from k-mers



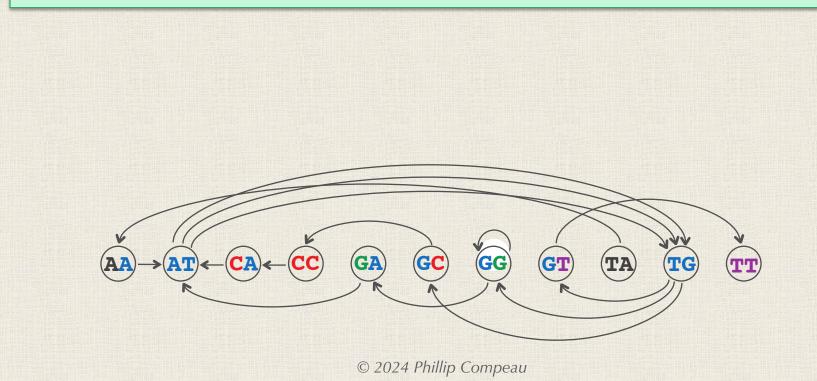


It's the Same Graph...



Approach for Constructing de Bruijn Graph

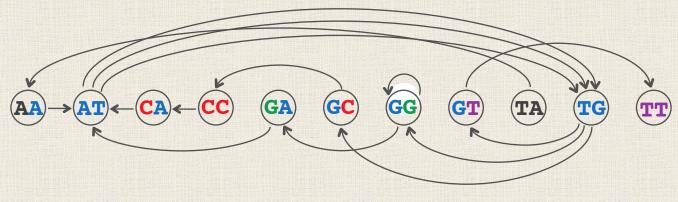
- 1. Form a node for every (k 1)-mer appearing as a prefix/suffix in *Patterns*.
- 2. For every string in *Patterns*, connect its prefix to its suffix.



Approach for Constructing de Bruijn Graph

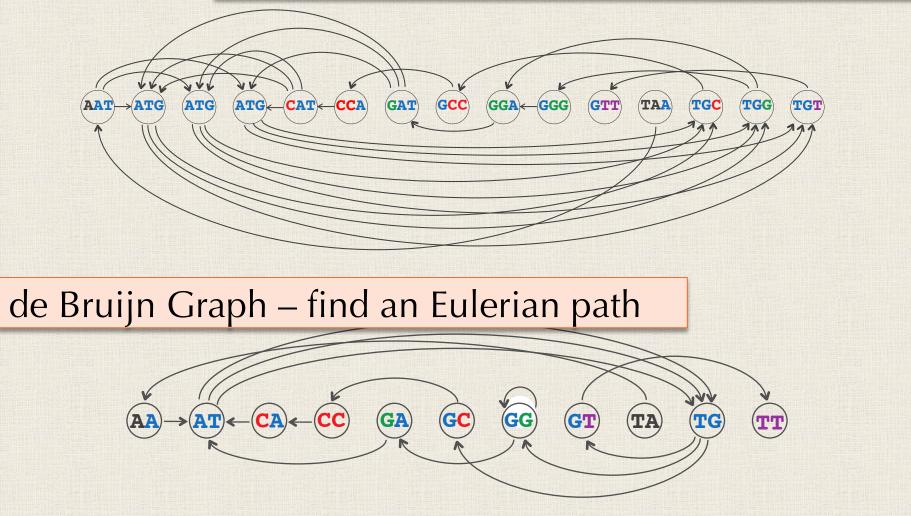
- 1. Form a node for every (k 1)-mer appearing as a prefix/suffix in *Patterns*.
- 2. For every string in *Patterns*, connect its prefix to its suffix.

STOP: Verify this approach for *Patterns* = {AAT ATG ATG ATG CAT CCA GAT GCC GGA GGG GTT TAA TGC TGG TGT}.



Which Graph Would You Rather Use?

Overlap Graph – find a Hamiltonian path

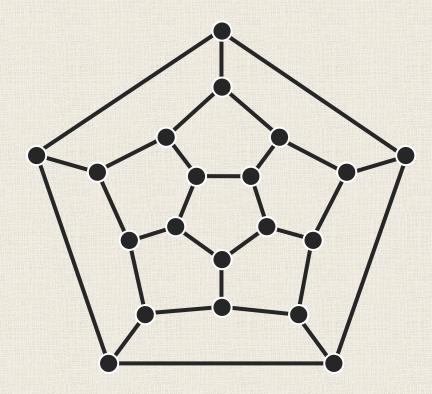


THE ICOSIAN GAME AND THE BRIDGES OF KONIGSBERG

The Origin of "Hamiltonian" Path/Cycle

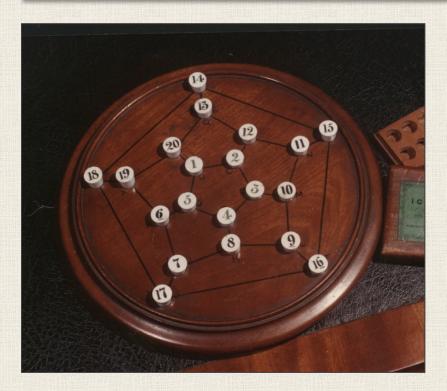
Hamiltonian cycle: A Hamiltonian path that returns to its starting node.

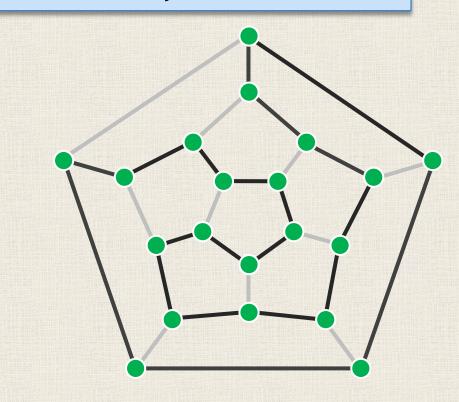
Exercise: Can you find a Hamiltonian cycle in this graph? (What algorithm did you use?)



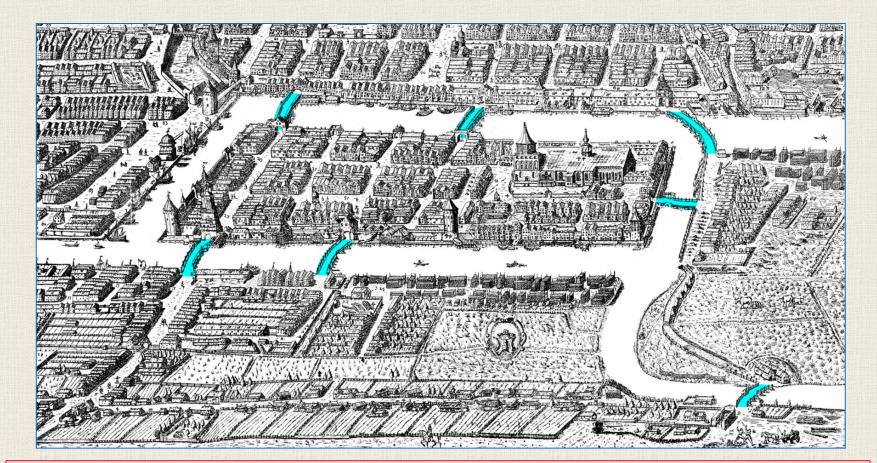
The Origin of "Hamiltonian" Path/Cycle

Icosian game: William *Hamilton,* 1857. Objective is to place pegs 1-20 one at a time in adjacent holes.





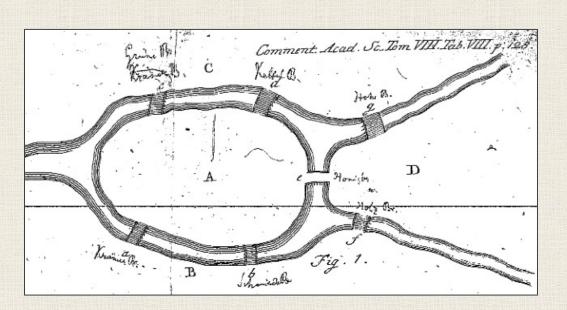
The Bridges of Königsberg



STOP: Is it possible to walk across each bridge *exactly once* and return to the starting point?

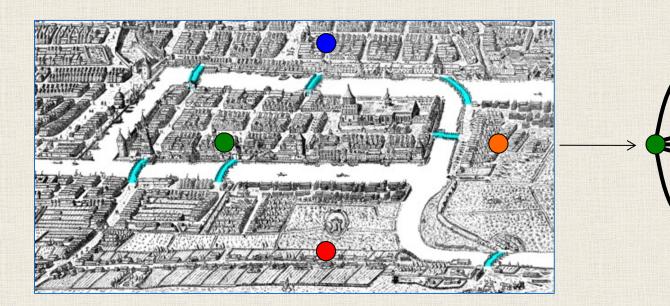
Define a graph:

- Nodes = 4 land masses
- Edges = 7 bridges



Define a graph:

- Nodes = 4 land masses
- Edges = 7 bridges

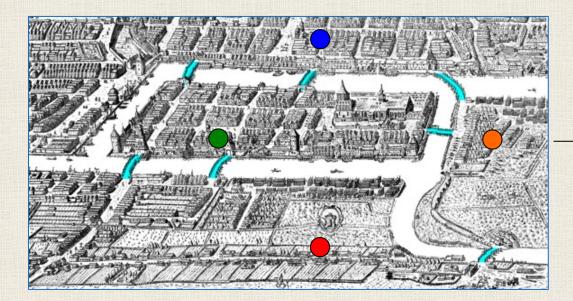


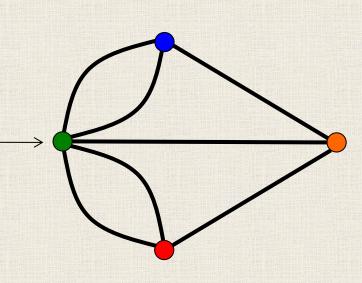
Note: The Bridges of Königsberg question has a solution when this graph has an *Eulerian* cycle.



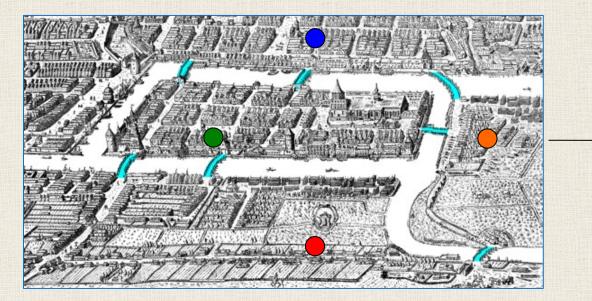


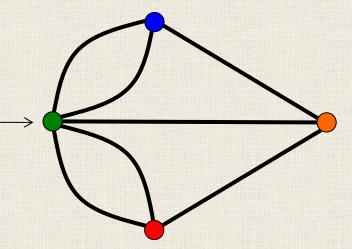
STOP: Does this graph help you solve the original question?





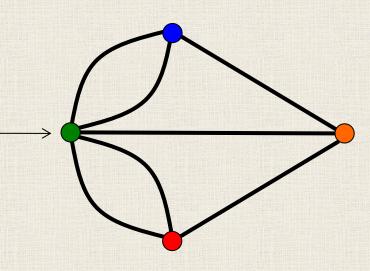
Answer: There is *no* solution because some nodes have an *odd* degree (number of incident edges).





Even better, Euler would *prove* how to quickly determine whether a graph has an Eulerian cycle.





Intractable Problems

Even better, Euler would *prove* how to quickly determine whether a graph has an Eulerian cycle.

Key Point: And yet no one has ever found a polynomial-time algorithm to find a Hamiltonian cycle in a graph!

Similar Problems with Different Fates

Hamiltonian Cycle Problem



Input: a network with *n* nodes. Output: "Yes" if there is a cycle visiting every node in the network; "No" otherwise.

Eulerian Cycle Problem

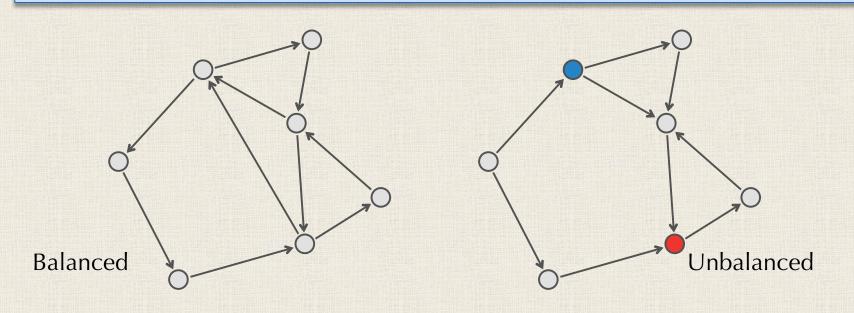


Input: a network with *n* nodes.
Output: "Yes" if there is a cycle visiting every edge in the network; "No" otherwise.

FROM EULER'S THEOREM TO AN ALGORITHM FOR GENOME ASSEMBLY

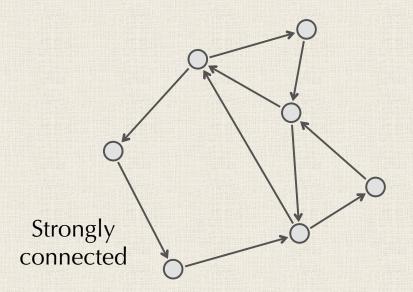
Euler's Theorem for Directed Graphs

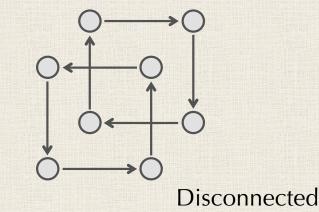
Indegree: Number of edges leading into a node. Outdegree: Number of edges leading out of a node. Balanced graph: Every node has indegree equal to outdegree.



Euler's Theorem for Directed Graphs

Strongly connected graph: A graph where it is possible to reach every node from any other node.

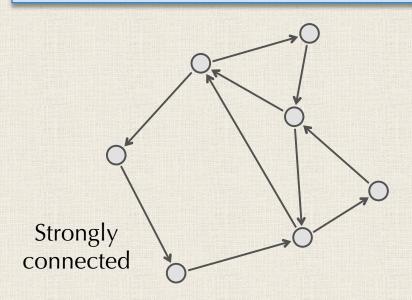


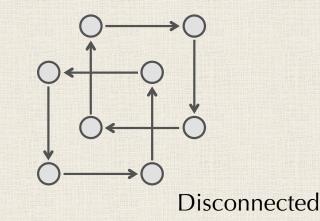


Euler's Theorem for Directed Graphs

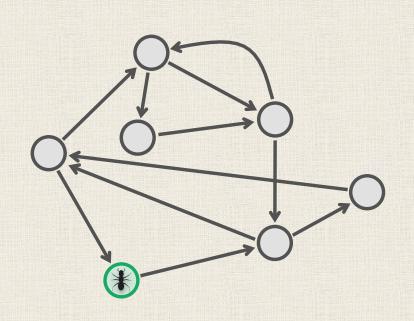
Strongly connected graph: A graph where it is possible to reach every node from any other node.

Euler's Theorem: Every balanced, strongly connected graph has an Eulerian cycle.

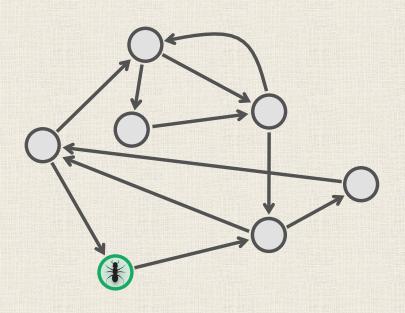




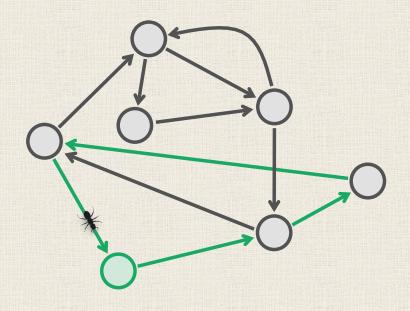
Take an arbitrary balanced, strongly connected network, place an ant on any starting node v_0 , and let it walk randomly.



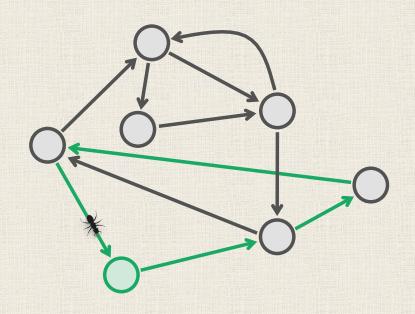
STOP: What must eventually happen when the ant "gets stuck"?



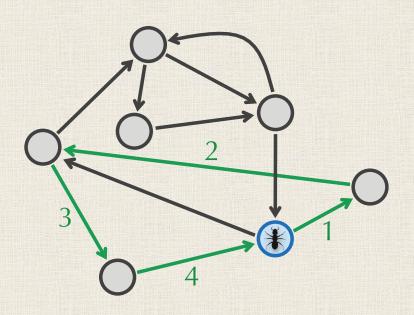
Answer: Because the graph is balanced, the ant must eventually get stuck at v_0 !



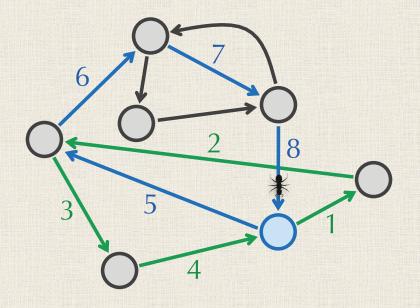
If this cycle, which we call $Cycle_0$, is Eulerian, then we stop. Otherwise, move the ant to a node on $Cycle_0$ that still has unused edges, called v_1 .



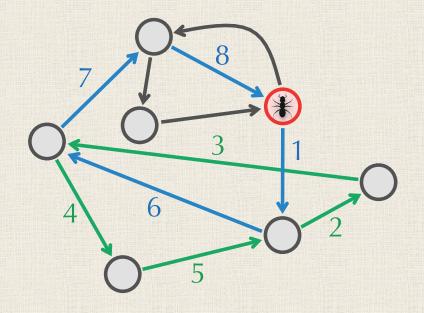
Make the ant traverse all of $Cycle_0$ first, then explore unused edges.



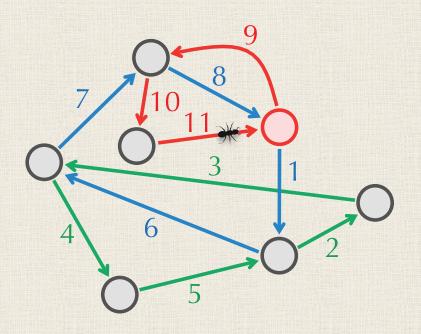
The same reasoning implies that the ant will eventually get stuck at v_1 , creating Cycle₁.



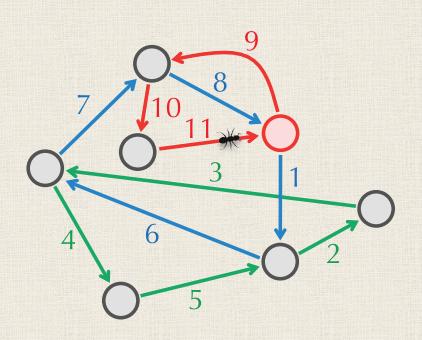
We simply iterate this procedure until we are out of unused edges, when we have an Eulerian cycle!



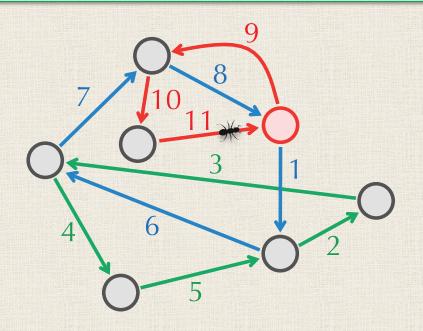
We simply iterate this procedure until we are out of unused edges, when we have an Eulerian cycle!



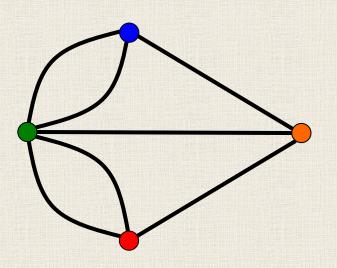
STOP: Why can we be sure that this process will use all the edges?



Answer: Because the graph is strongly connected! So note that we have used both conditions in the theorem (balanced and strongly connected).



Exercise: When will an "undirected" graph have an Eulerian cycle?

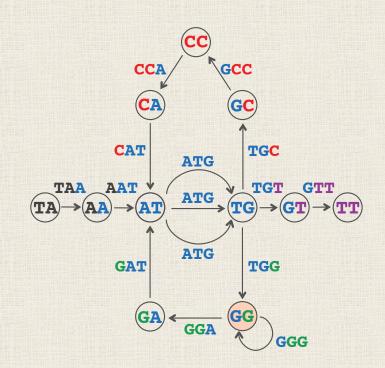


Euler's Theorem is "Constructive"

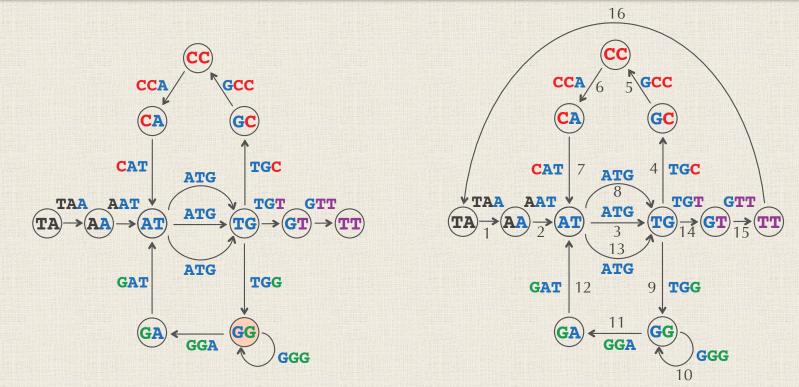
Key Point: This is a "constructive proof", meaning it implies an algorithm for finding an Eulerian cycle.

EulerianCycle(Graph) $v \leftarrow$ arbitrary node in GraphCycle \leftarrow randomly walk starting at v (don't revisit edges) until cyclewhile there are unexplored edges in GraphnewStart \leftarrow node in Cycle with unexplored edgesCycle' \leftarrow cycle formed by traversing Cycle (starting at newStart)and then randomly walkingCycle \leftarrow Cycle'return Cycle

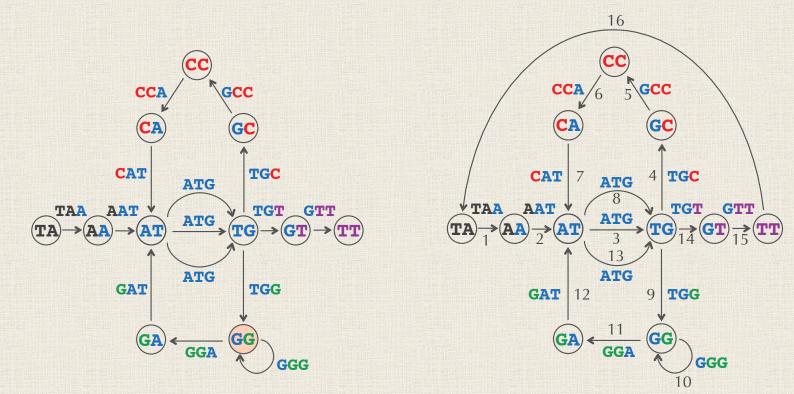
STOP: How do we find an Eulerian *path* in this graph?



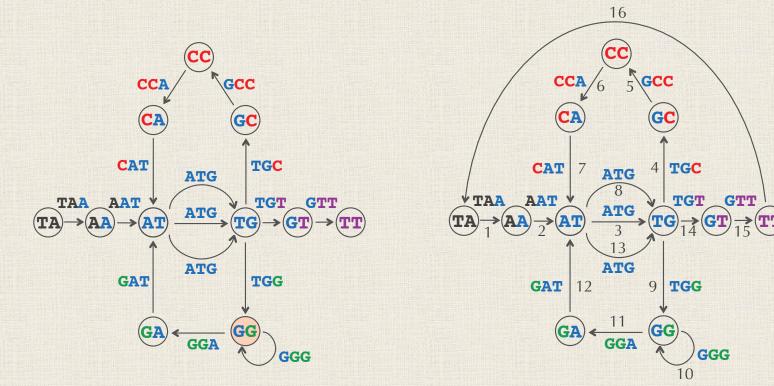
Answer: Simply draw an edge connecting the two unbalanced nodes to form a balanced graph. Eulerian cycle on right = Eulerian path on left.



STOP: Why will the augmented de Bruijn graph on the right be balanced for any collection of strings *Patterns*?



Answer: For every node *v* in de Bruijn graph, Indegree(*v*) and Outdegree(*v*) are both equal to # of patterns containing *v* as prefix/suffix, respectively.



String Reconstruction Problem: *Reconstruct a string from its k-mer composition.*

Input: An integer *k* and a collection *Patterns* of *k*-mers.

Output: A string *Text* with *k*-mer composition equal to *Patterns* (if such a string exists).

1. Form de Bruijn graph *G* from *Patterns*.

String Reconstruction Problem: *Reconstruct a string from its k-mer composition.*

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Form de Bruijn graph *G* from *Patterns*.
 Add edge to make modified graph *G'* balanced.

String Reconstruction Problem: *Reconstruct a string from its k-mer composition.*

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1. Form de Bruijn graph *G* from *Patterns*.

Add edge to make modified graph G' balanced.
 Find Eulerian cycle in G'.

String Reconstruction Problem: *Reconstruct a string from its k-mer composition.*

Input: An integer *k* and a collection *Patterns* of *k*-mers.

Output: A string *Text* with *k*-mer composition equal to *Patterns* (if such a string exists).

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- 5. Convert "genome path" into string Text.

Aside: De Bruijn/Good's Question

Recall: a binary string is *k*-universal if it contains every binary *k*-mer once.

STOP: How can we find a *k*-universal binary string?



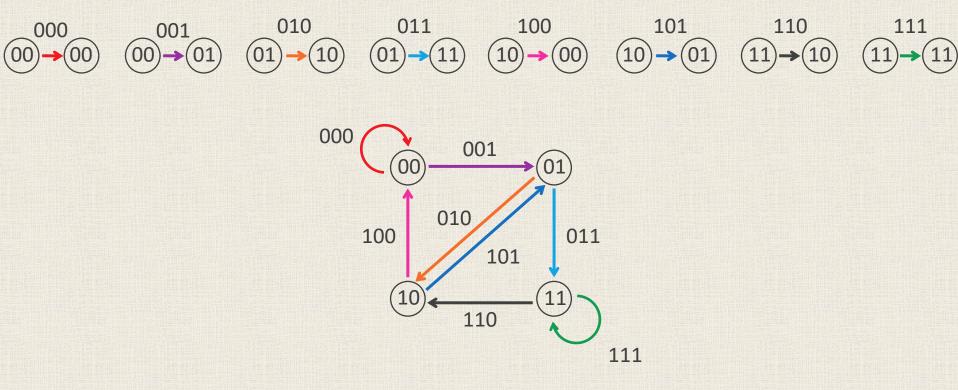
Jack Good



Nicolaas de Bruijn

Aside: De Bruijn/Good's Question

Answer: Construct the "de Bruijn graph" for *Patterns* = all binary *k*-mers; find Eulerian path.



DE BRUIJN GRAPHS FACE HARSH PRACTICAL REALITIES

Practical Sequencing Complications

1. DNA may be divided over **multiple chromosomes**.

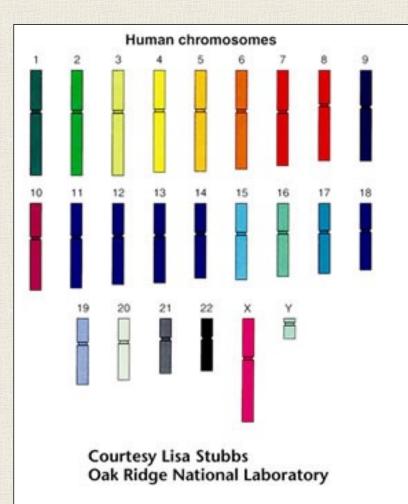
2. Reads have **imperfect "coverage"** of the underlying genome – there may be some regions that are not covered by any reads.

3. Sequencing machines are error-prone.

4. DNA is **double-stranded**.

Genomes May Have Multiple Chromosomes

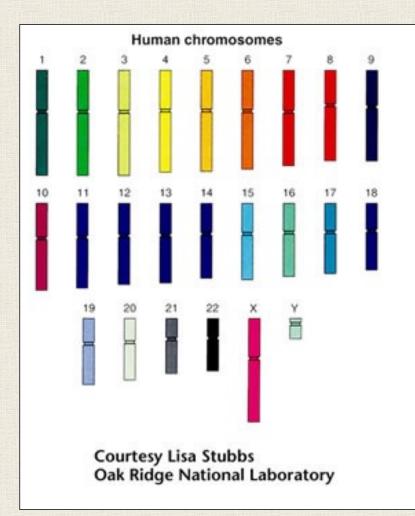
STOP: Any ideas for assembling a genome with multiple chromosomes?



Genomes May Have Multiple Chromosomes

STOP: Any ideas for assembling a genome with multiple chromosomes?

Answer: In theory, we just find an Eulerian path in *n* different de Bruijn graphs...



Read Coverage is Never Perfect



ATGCCGTATGGACAACGACT ATGCCGTATG GCCGTATGGA GTATGGACAA GACAACGACT

Note that these reads don't overlap perfectly, so building a de Bruijn graph will fail.

ATGCCGTATGGACAACGACT ATGCCGTATG GCCGTATGGA GTATGGACAA GACAACGACT ATGCCGTATGGACAACGACT ATGCC TGCCG GCCGT CGTAT GTATG

Read breaking: Split each read into all its *k*-mer substrings (for a smaller value of *k*).

ATGCCGTATGGACAACGACT ATGCCGTATG GCCGTATGGA GTATGGACAA GACAACGACT ATGCCGTATGGACAACGACT ATGCC TGCCG GCCGT CGTAT GTATG TATGG ATGGA

ATGCCGTATGGACAACGACT ATGCCGTATG GCCGTATGGA GTATGGACAA GACAACGACT ATGCCGTATGGACAACGACT ATGCC GCCGT GCCGT CGTAT GTATG GTATG ATGGA TGGAC GGACA GACAA

ATGCCGTATGGACAACGACT ATGCCGTATG GCCGTATGGA GTATGGACAA GACAACGACT

ATGCCGTATGGACAACGACT ATGCC TGCCG GCCGT CGTAT GTATG TATGG ATGGA TGGAC GGACA GACAA ACAAC CAACG AACGA ACGAC

ATGCCGTATGGACAACGACT ATGCCGTATG GCCGTATGGA GTATGGACAA GACAACGACT

ATGCCGTATGGACAACGACT ATGCC TGCCG GCCGT CGTAT GTATG TATGG ATGGA TGGAC GGACA GACAA ACAAC CAACG AACGA ACGAC

CGACT

Boosting Coverage through Read Breaking

ATGCCGTATGGACAACGACT ATGCCGTATG GCCGTATGGA GTATGGACAA GACAACGACT

ATGCCGTATGGACAACGACT ATGCC TGCCG GCCGT CGTAT GTATG TATGG ATGGA TGGAC GGACA GACAA ACAAC CAACG AACGA ACGAC

CGACT

STOP: What are the trade-offs in choosing a value of *k*?

Boosting Coverage through Read Breaking

ATGCCGTATGGACAACGACT ATGCCGTATG GCCGTATGGA GTATGGACAA GACAACGACT

ATGCCGTATGGACAACGACT ATGCC TGCCG GCCGT CGTAT GTATG TATGG ATGGA TGGAC GGACA GACAA ACAAC CAACG AACGA ACGAC

Answer: The smaller the value of *k*, the higher our coverage will be, but also the more repeats and the more "tangled" our graph.

Assembling Contigs

Even after read breaking, most assemblies have gaps in their coverage, and we will not have a true Eulerian path in the de Bruijn graph.

Assembling Contigs

Even after read breaking, most assemblies have gaps in their coverage, and we will not have a true Eulerian path in the de Bruijn graph.

Real assembly software instead tries to infer (a small number of) **contigs:** contiguous genome segments.

Genome —				
Reads	_	<u> </u>		
Contigs	¥	v	Ÿ	Ÿ

Contigs Lurking in the de Bruijn Graph

A path in a graph is called **non-branching** if InDegree(v) = OutDegree(v) = 1 for each "intermediate" node v in the path.

Contigs Lurking in the de Bruijn Graph

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A maximal non-branching path is a non-branching path that cannot made longer in either direction.

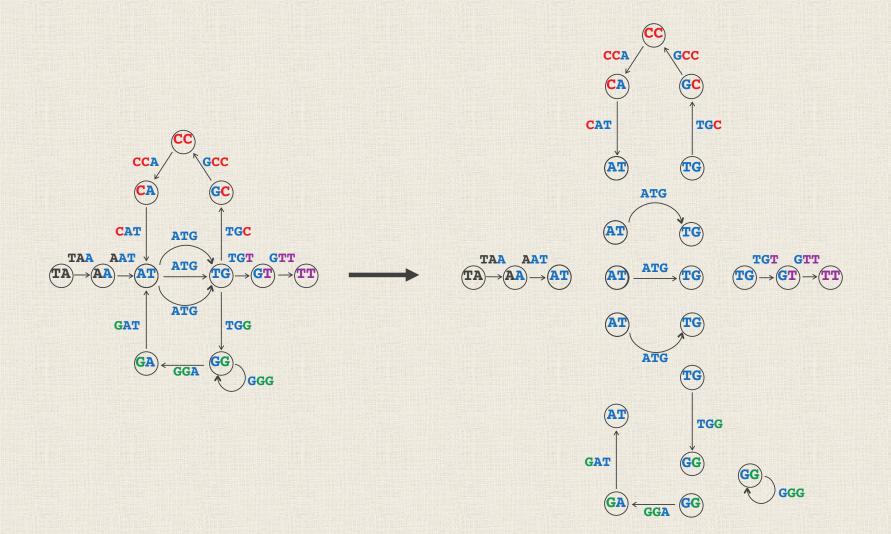
Contigs Lurking in the de Bruijn Graph

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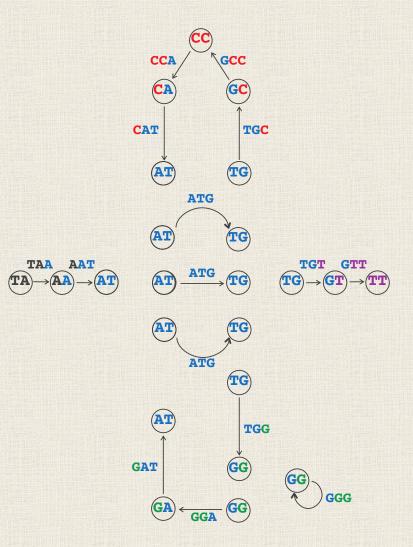
Note: In mathematics, "maximum" means "global maximum"; "maximal" means "local maximum".

Transforming dB Graph into Paths



Transforming dB Graph into Paths

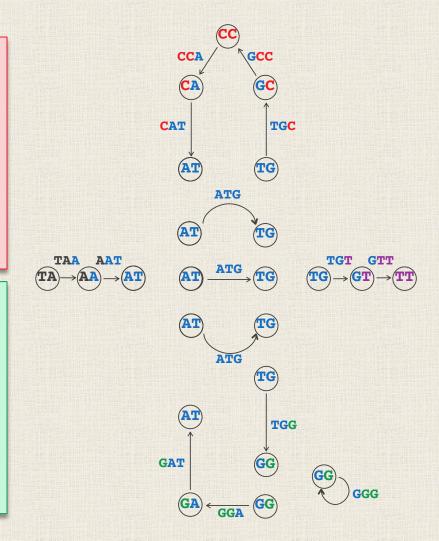
STOP: Why do you think we are interested in maximal non-branching paths in genome assembly?



Transforming dB Graph into Paths

STOP: Why do you think we are interested in maximal non-branching paths in genome assembly?

Answer: They represent "subpaths" that must be present in *any* assembly, and so we can be confident in them.



Assembling Error-Prone Reads

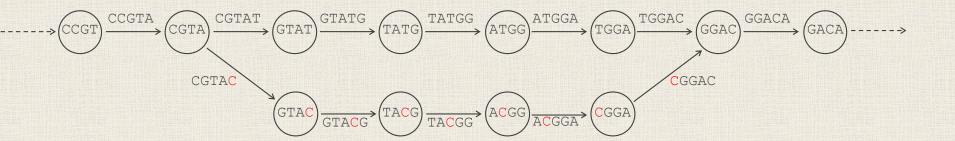
STOP: Say we sequence both the correct read CGTATGGACA and the incorrect read CGTACGGACA. What will we see in the de Bruijn graph after read breaking for k = 5?

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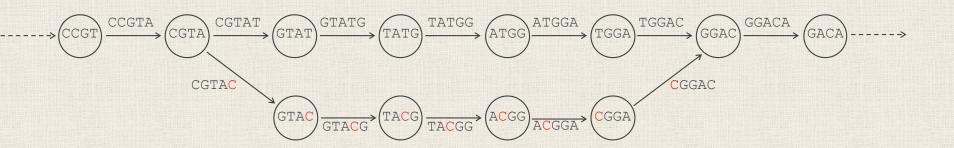
Answer	r: A "bubble"!	
\rightarrow (CCGT) $\xrightarrow{\text{CCGTA}}$ (C	$CGTA \xrightarrow{CGTAT} GTAT \xrightarrow{GTATG} TATG \xrightarrow{TATGG} ATGG \xrightarrow{ATGGA} TGGAC \xrightarrow{GGAC} GGAC \xrightarrow{GGACA} GACA \xrightarrow{GGAC} GACA \xrightarrow{GAC} GAC$	>
	CGTAC GTAC GTACG GTACG TACGG TACG T	

Bubble: Two disjoint short path (less than some threshold length) connecting the same pair of nodes in the de Bruijn graph.

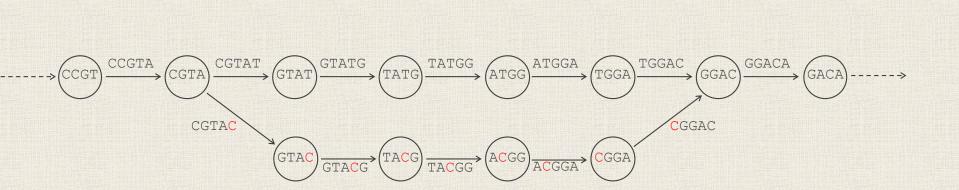


Bubble: Two disjoint short path (less than some threshold length) connecting the same pair of nodes in the de Bruijn graph.

STOP: How might we remove bubbles? What would cause your approach to go wrong?

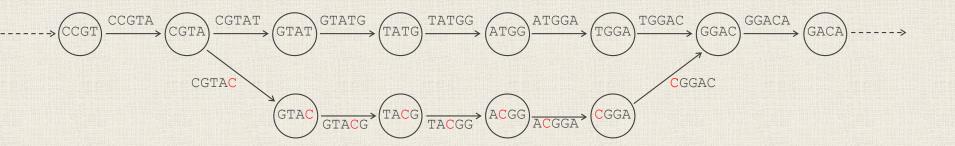


Inexact repeat: Repeated region in genome with minor variations; the variations look just like sequencing errors!

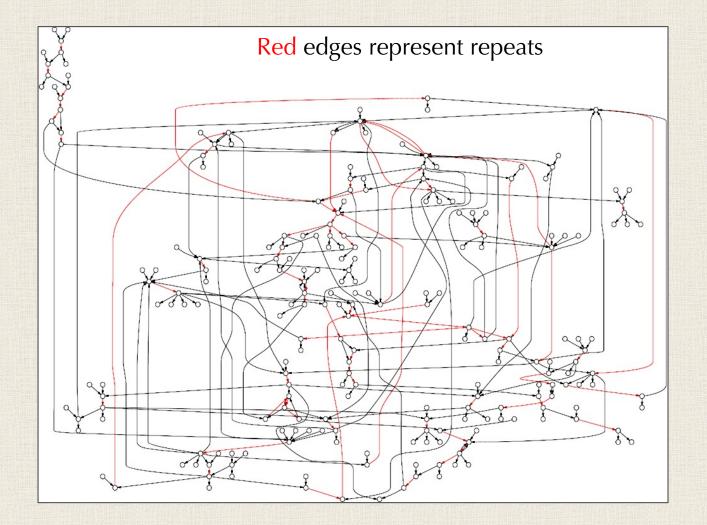


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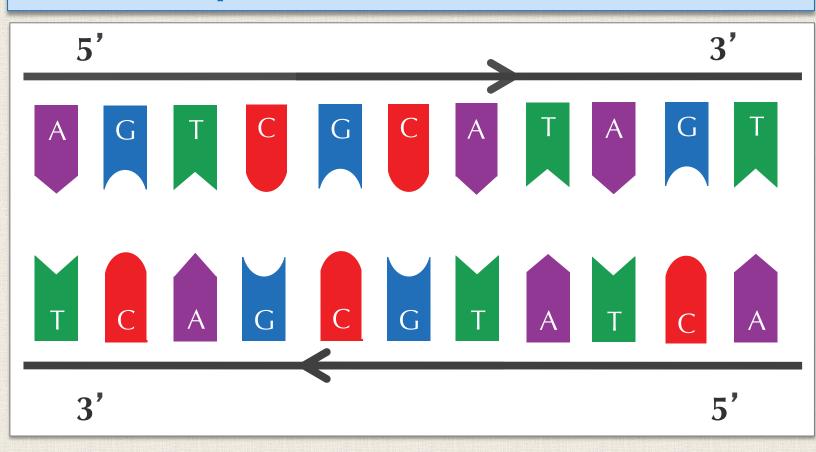
Lower "multiplicity" paths are likely errors; this is one more benefit of higher coverage in assembly.



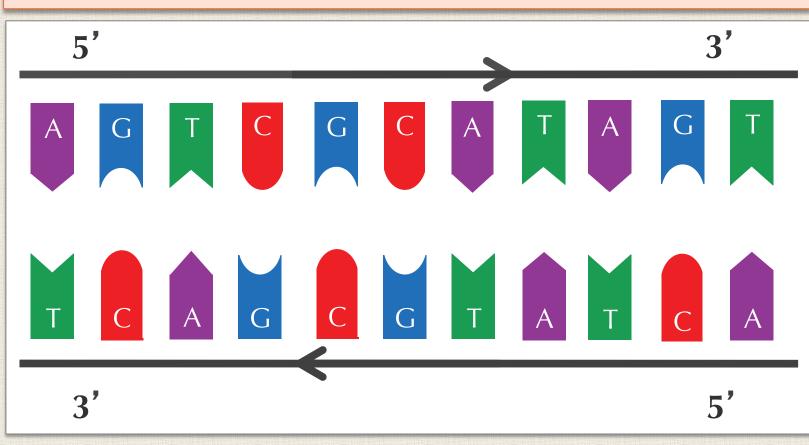
dB Graph of N. *meningitidis* (Bacterium) *After* Removing Bubbles



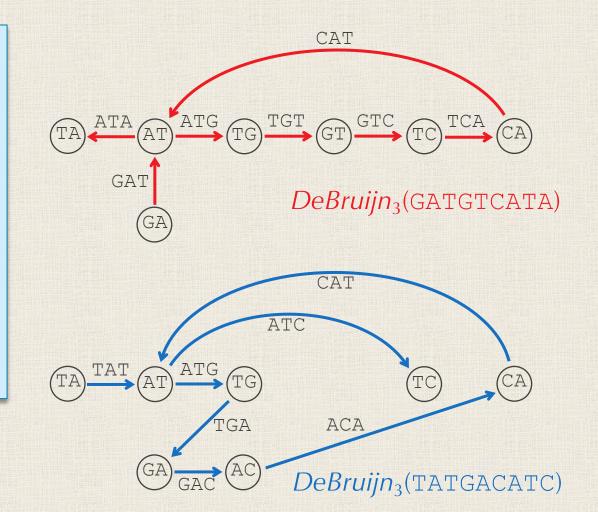
DNA is double-stranded, and the two strands are **reverse complements** of each other.



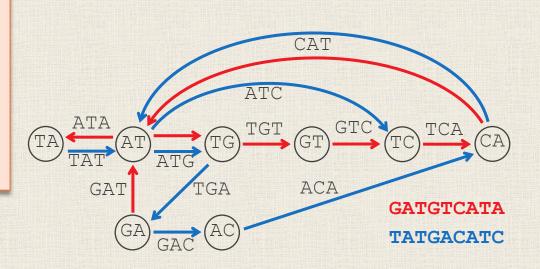
Reads may come from *either strand*, so we need to consider each read's reverse complement.



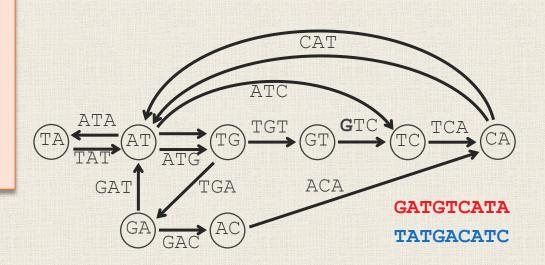
Note that this example is trivial if we had two de Bruijn graphs (one for the string, one for its reverse complement).



The reality is that we see the amalgamation of both graphs.



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Even though neither string has a repeat, the graph becomes tangled because ATG and CAT are **inverted repeats:** the strings are reverse complements of each other.

de Bruijn Assembly in Real Research

An Eulerian path approach to DNA fragment assembly | PNAS

Our main result is the reduction of the **fragment assembly** to a variation of the classical **Eulerian path** problem that allows one to generate accurate solutions of large-scale sequencing problems. ... For the last 20 years, **fragment assembly** in **DNA** sequencing mainly followed the "overlap–layout–consensus" paradigm (1–6).

by PA Pevzner · 2001 · Cited by 1522 · Related articles

Velvet: algorithms for de novo short read assembly using de Bruijn graphs DR Zerbino, <u>E Birney</u> - Genome research, 2008 - genome.cshlp.org

... set of **algorithms**, collectively named "**Velvet**,... **algorithm** merges sequences that belong together, then the repeat solver separates paths sharing local overlaps. We have assessed **Velvet** ... \therefore Save \Im Cite Cited by 10928 Related articles All 24 versions Web of Science: 7291

SPAdes: a new genome assembly **algorithm** and its applications to single-cell sequencing

A Bankevich, S Nurk, D Antipov... - Journal of ..., 2012 - liebertpub.com

... We present the **SPAdes** assembler, introducing a number of ..., the basis of many fragment assembly **algorithms**. However, a ... Unfortunately, while there is a simple **algorithm** for the former ... \therefore Save 55 Cite Cited by 20780 Related articles All 14 versions Web of Science: 15404